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PGT(MATHEMATICS)

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(AHMEDABAD CLUSTER)

SUPPORT MATERIAL

CLASS: X SESSION 2020-21

COURSE STRUCTURE

MATHEMATICS (CLASS–X)

UNITS	UNIT NAME	MARKS
I.	NUMBER SYSTEMS	06
II.	ALGEBRA	20
III.	COORDINATE GEOMETRY	06
IV.	GEOMETRY	15
V.	TRIGONOMETRY	12
VI.	MENSURATION	10
VII.	STATISTICS & PROBABILITY	11
	TOTAL	80

DELETED SYLLABUS

CHAPTER	TOPICS REMOVED
UNIT I-NUMBER SYSTEMS	
REAL NUMBERS	Euclid's division lemma
UNIT II-ALGEBRA	
POLYNOMIALS	Statement and simple problems on division algorithm for polynomials with real coefficients.
PAIR OF LINEAR EQUATIONS IN TWO VARIABLES	cross multiplication method
QUADRATIC EQUATIONS	Situational problems based on equations **reducible to quadratic equations**
ARITHMETIC PROGRESSIONS	Application in solving daily life problems **based on sum to n terms**
UNIT III-COORDINATE GEOMETRY	
COORDINATE GEOMETRY	Area of a triangle.

UNIT IV-GEOMETRY	
TRIANGLES	Proof of the following theorems are deleted • The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. • In a triangle, if the square on one side is equal to sum of the squares on the other two sides, the angle opposite to the first side is a right angle.
CIRCLES	No deletion
CONSTRUCTIONS	• Construction of a triangle similar to a given triangle.
UNIT V- TRIGONOMETRY	
INTRODUCTION TO TRIGONOMETRY	• motivate the ratios whichever are defined at 0° and 90°
TRIGONOMETRIC IDENTITIES	• Trigonometric ratios of complementary angles.
HEIGHTS AND DISTANCES	No deletion
UNIT VI-MENSURATION	
AREAS RELATED TO CIRCLES	• Problems on central angle of 120°

UNIT VI-MENSURATION	
AREAS RELATED TO CIRCLES	• Problems on central angle of 120°

SURFACE AREAS AND VOLUMES	• Frustum of a cone.
UNIT VI-STATISTICS & PROBABILITY	
STATISTICS	• Step deviation Method for finding the mean • Cumulative Frequency graph
PROBABILITY	No deletion

NOTES ON THE CHAPTERS

CHAPTER 1

R = Real Numbers:
All rational and irrational numbers are called real numbers.

I = Integers:
All numbers from (...-3, -2, -1, 0, 1, 2, 3...) are called integers.

Q = Rational Numbers:
Real numbers of the form $\frac{p}{q}$, q ≠ 0, p, q ∈ I are rational numbers.

- All integers can be expressed as rational, for example, 5 = $\frac{5}{1}$
- Decimal expansion of rational numbers terminating or non-terminating recurring.

Q' = Irrational Numbers:
Real numbers which cannot be expressed in the form $\frac{p}{q}$ and whose decimal expansions are non-terminating and non-recurring.

- Roots of primes like √2, √3, √5 etc. are irrational

N = Natural Numbers:
Counting numbers are called natural numbers. N = {1, 2, 3, ...}

W = Whole Numbers:
Zero along with all natural numbers are together called whole numbers. {0, 1, 2, 3,...}

Even Numbers:
Natural numbers of the form 2n are called even numbers. (2, 4, 6, ...}

Odd Numbers:
Natural numbers of the form 2n -1 are called odd numbers. {1, 3, 5, ...}

- Why can't we write the form as 2n+1?

Remember this!

- All Natural Numbers are whole numbers.
- All Whole Numbers are Integers.

- All Integers are Rational Numbers.
- All Rational Numbers are Real Numbers.

Prime Numbers:

The natural numbers greater than 1 which are divisible by 1 and the number itself are called prime numbers, Prime numbers have two factors i.e., 1 and the number itself For example, 2, 3, 5, 7 & 11 etc.

- 1 is not a prime number as it has only one factor.

Composite Numbers:

The natural numbers which are divisible by 1, itself and any other number or numbers are called composite numbers. For example, 4, 6, 8, 9, 10 etc.

Note: 1 is neither prime nor a composite number.

1. Algorithm to locate HCF and LCM of two or more positive integers:

Step I:

Factorize each of the given positive integers and express them as a product of powers of primes in ascending order of magnitude of primes.

Step II:

To find HCF, identify common prime factor and find the least powers and multiply them to get HCF.

Step III:

To find LCM, find the greatest exponent and then multiply them to get the LCM.

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2. To prove Irrationality of numbers:

- The sum or difference of a rational and an irrational number is irrational.
- The product or quotient of a non-zero rational number and an irrational number is irrational.

3. To determine the nature of the decimal expansion of rational numbers:

- Let x = p/q, p and q are co-primes, be a rational number whose decimal expansion terminates. Then the prime factorization of 'q' is of the form $2^m 5^n$, m and n are non-negative integers.
- Let x = p/q be a rational number such that the prime factorization of 'q' is not of the form $2^m 5^n$, 'm' and 'n' being non-negative integers, then x has a non-terminating repeating decimal expansion.

CHAPTER 2

- "Polynomial" comes from the word 'Poly' (Meaning Many) and 'nomial' (in this case meaning Term)-so it means many terms.
- A polynomial is made up of terms that are only added, subtracted or multiplied.
- A quadratic polynomial in x with real coefficients is of the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.
- Degree – The highest exponent of the variable in the polynomial is called the degree of polynomial. Example: $3x^3 + 4$, here degree = 3.
- Polynomials of degrees 1, 2 and 3 are called linear, quadratic and cubic polynomial respectively.
- A polynomial can have terms which have Constants like 3, -20, etc., Variables like x and y and Exponents like 2 in y^2.
- These can be combined using addition, subtraction and multiplication but NOT DIVISION.
- The zeroes of a polynomial p(x) are precisely the x-coordinates of the points, where the graph of y = p(x) intersects the x-axis.

If α and β are the zeroes of the quadratic polynomial $ax^2 + bx + c$, then

$$sum \quad of \quad zeros, \alpha + \beta = \frac{-b}{a} = \frac{-coefficient \quad of \quad x}{coefficient \quad of \quad x^2}$$

$$product \quad of \quad zeros, \alpha\beta = \frac{c}{a} = \frac{constant \quad term}{coefficient \quad of \quad x^2}$$

If α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d = 0$, then

$$\alpha+\beta+\gamma = \frac{-b}{a} = \frac{-coefficient \quad of \quad x^2}{coefficient \quad of \quad x^3}$$

$$\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a} = \frac{coefficient \quad of \quad x}{coefficient \quad of \quad x^3}$$

$$\alpha\beta\gamma = \frac{-d}{a} = \frac{-constant \quad term}{coefficient \quad of \quad x^3}$$

Zeroes (α, β, γ) follow the rules of algebraic identities, i.e.,

$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\therefore (\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta$

CHAPTER 3

- For any linear equation, each solution (x, y) corresponds to a point on the line. General form is given b $ax + by + c = 0$.
- The graph of a linear equation is a straight line.
- Two linear equations in the same two variables are called a pair of linear equations in two variables. The most general form of a pair of linear equations is: $a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$ where a_1, a_2, b_1, b_2, c_1 and c_2 are real numbers, such that $a_1^2 + b_1^2 \neq 0$, $a_2^2 + b_2^2 \neq 0$.
- A pair of values of variables 'x' and 'y' which satisfy both the equations in the given system of equations is said to be a solution of the simultaneous pair of linear equations.
- A pair of linear equations in two variables can be represented and solved, by
  (i) Graphical method
  (ii) Algebraic method

(i) Graphical method. The graph of a pair of linear equations in two variables is presented by two lines.

(ii) Algebraic methods. Following are the methods for finding the solutions(s) of a pair of linear equations:

1. Substitution method
2. Elimination method

- There are several situations which can be mathematically represented by two equations that are not linear to start with. But we allow them so that they are reduced to a pair of linear equations.
- **Consistent system.** A system of linear equations is said to be consistent if it has at least one solution
- **Inconsistent system.** A system of linear equations is said to be inconsistent if it has no solution.

CONDITIONS FOR CONSISTENCY

Let the two equations be:

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Then,

Relationship between coeff. or the pair of equations	Graph	Number of Solutions	Consistency of System
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Unique solution	Consistent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution	Inconsistent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Co-incident lines	Infinite solutions	Consistent

CHAPTER 4

A quadratic polynomial of the form $ax^2 + bx + c$, where $a \neq 0$ and a, b, c are real numbers, is called a quadratic equation
when $ax^2 + bx + c = 0$.
Here a and b are the coefficients of x^2 and x respectively and 'c' is a constant term.

Any value is a solution of a quadratic equation if and only if it satisfies the quadratic equation.

Quadratic formula: The roots, i.e., α and β of a quadratic equation $ax^2 + bx + c = 0$ are given by $\frac{-b\pm\sqrt{D}}{2a}$ or $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ provided $b^2 - 4ac \geq 0$.

Here, the value $b^2 - 4ac$ is known as the discriminant and is generally denoted by D. 'D' helps us to determine the nature of roots for a given quadratic equation. Thus $D = b^2 - 4ac$.

The rules are:

1. If D = 0 ⇒ The roots are Real and Equal.
2. If D > 0 ⇒ The two roots are Real and Unequal.
3. If D < 0 ⇒ No Real roots exist.

If α and β are the roots of the quadratic equation, then Quadratic equation is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ Or $x^2 -$ (sum of roots) x + product of roots = 0

where, Sum of roots (α + β) = $\frac{-coefficient \quad of \quad x}{coefficient \quad of \quad x^2} = \frac{-b}{a}$

Product of roots (α x β) = $\frac{coefficient \quad term}{coefficient \quad of \quad x^2} = \frac{c}{a}$

CHAPTER 5

SEQUENCE:

A sequence is an arrangement of numbers in a definite order and according to some rule.

Example: 1, 3, 5,7,9, ... is a sequence where each successive item is 2 greater than the preceding term and 1, 4, 9, 16, 25, ... is a sequence where each term is the square of successive natural numbers.

TERMS :

The various numbers occurring in a sequence are called 'terms'. Since the order of a sequence is fixed, therefore the terms are known by the position they occupy in the sequence.

Example: If the sequence is defined as

1	3	5	7	9	..	n	
↓	↓	↓				↓	
First term (a_1)	Second term (a_2)	Third term (a_3)				n^{th} term (a_n)	or general term

ARITHMETIC PROGRESSION (A.P.):

An Arithmetic progression is a special case of a sequence, where the difference between a term and its preceding term is always constant, known as common difference, i.e., d. The arithmetic progression is abbreviated as A.P.

The general form of an A.P. is
$\therefore$ a, a + d, a + 2d,... For example, 1, 9, 11, 13.., Here the common difference is 2. Hence it is an A.P.

In an A.P. with first term a and common difference d, the nth term (or the general term) is given by .
$a_n = a + (n - 1)d$.
...where [a = first term, d = common difference, n = term number
Example: To find seventh term put n = 7
$\therefore a_7 = a + (7 - 1)d$ or $a_7 = a + 6d$

The sum of the first n terms of an A.P. is given by
$S_n = \frac{n}{2}[2a + (n - 1)d]$ or $\frac{n}{2}[a + 1]$
where, 1 is the last term of the finite AP.

If a, b, c are in A.P. then b = $\frac{a+c}{2}$ and b is called the arithmetic mean of a and c.

CHAPTER 6

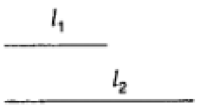
SIMILAR FIGURES

- Two figures having the same shape but not necessary the same size are called similar figures.
- All congruent figures are similar but all similar figures are not congruent.

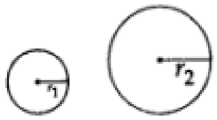
SIMILAR POLYGONS

Two polygons are said to be similar to each other, if:
(i) their corresponding angles are equal, and
(ii) the lengths of their corresponding sides are proportional

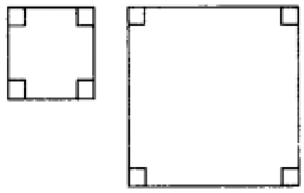
Example:
Any two line segments are similar since length are proportional



Any two circles are similar since radii are proportional



Any two squares are similar since corresponding angles are equal and lengths are proportional.

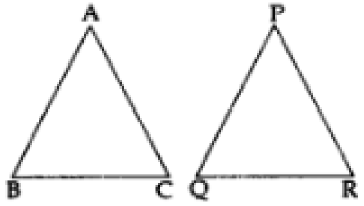


Note:

Similar figures are congruent if there is one to one correspondence between the figures.

∴ From above we deduce:

Any two triangles are similar, if their



(i) Corresponding angles are equal

∠A = ∠P

∠B = ∠Q

∠C = ∠R

(ii) Corresponding sides are proportional

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$$

THALES THEOREM OR BASIC PROPORTIONALITY THEORY

Theorem 1:

State and prove Thales' Theorem.

CRITERION FOR SIMILARITY OF TRIANGLES

Two triangles are similar if either of the following three criterion's are satisfied:

- **AAA similarity Criterion.** If two triangles are equiangular, then they are similar.
- **Corollary(AA similarity).** If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
- **SSS Similarity Criterion.** If the corresponding sides of two triangles are proportional, then they are similar.
- **SAS Similarity Criterion.** If in two triangles, one pair of corresponding sides are proportional and the included angles are equal, then the two triangles are similar.

AREA OF SIMILAR TRIANGLES

Theorem 2.

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Results based on Area Theorem:

1. Ratio of areas of two similar triangles = Ratio of squares of corresponding altitudes
2. Ratio of areas of two similar triangles = Ratio of squares of corresponding medians
3. Ratio of areas of two similar triangles = Ratio of squares of corresponding angle bisector segments.

Note:

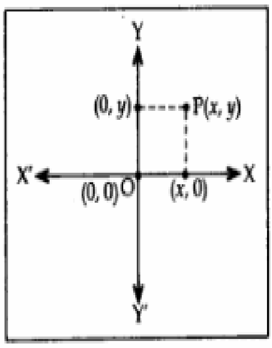
If the areas of two similar triangles are equal, the triangles are congruent.

PYTHAGORAS THEOREM

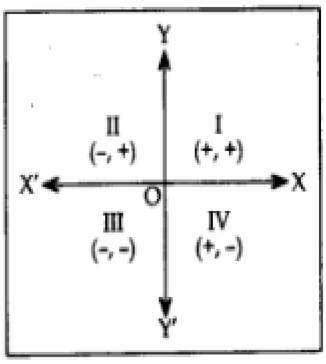
Theorem 3:

State and prove Pythagoras' Theorem.

- Position of a point P in the Cartesian plane with respect to co-ordinate axes is represented by the ordered pair (x, y).



- The line X'OX is called the X-axis and YOY' is called the Y-axis.
- The part of intersection of the X-axis and Y-axis is called the origin O and the co-ordinates of O are (0, 0).
- The perpendicular distance of a point P from the Y-axis is the 'x' co-ordinate and is called the abscissa.
- The perpendicular distance of a point P from the X-axis is the 'y' co-ordinate and is called the ordinate.
- Signs of abscissa and ordinate in different quadrants are as given in the diagram:



- Any point on the X-axis is of the form (x, 0).
- Any point on the Y-axis is of the form (0, y).
- The distance between two points P(x1, y1) and Q (x2, y2) is given by

  PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

  Note. If O is the origin, the distance of a point P(x, y) from the origin O(0, 0) is given by

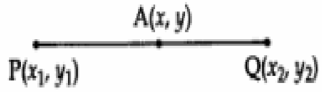
  OP = $\sqrt{x^2 + y^2}$

Section formula. The coordinates of the point which divides the line segment joining the points A(x1, y1) and B(x2, y2) internally in the ratio m : n are:

$$P(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

The above formula is section formula. The ratio m: n can also be written as $\frac{m}{n}$: 1 or k : 1, The co-ordinates of P can also be written as P(x,y) = $\frac{kx_2+x_1}{k+1}, \frac{ky_2+y_1}{k+1}$

The mid-point of the line segment joining the points P(x1, y1) and Q(x2, y2) is



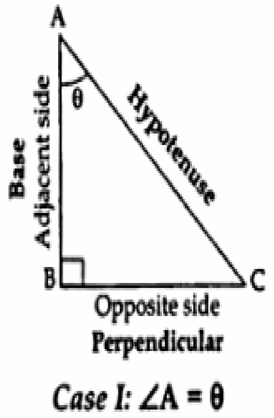
$$A(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

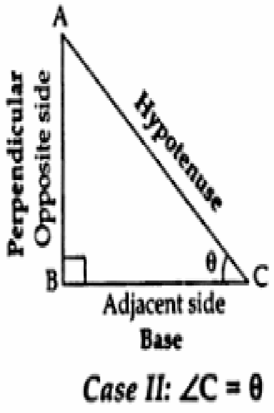
Here m : n = 1 :1.

CHAPTER 8

- Position of a point P in the Cartesian plane with respect to co-ordinate axes is represented by the ordered pair (x, y).
- Trigonometry is the science of relationships between the sides and angles of a right-angled triangle.
- Trigonometric Ratios: Ratios of sides of right triangle are called trigonometric ratios.

  Consider triangle ABC right-angled at B. These ratios are always defined with respect to acute angle 'A' or angle 'C.
- If one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of an angle can be easily determined.
- How to identify sides: Identify the angle with respect to which the t-ratios have to be calculated. Sides are always labelled with respect to the 'θ' being considered.





In a right triangle ABC, right-angled at B. Once we have identified the sides, we can define six t-Ratios with respect to the sides.

case I	case II
(i) sine A = $\frac{perpendicular}{hypotenuse} = \frac{BC}{AC}$	(i) sine C = $\frac{perpendicular}{hypotenuse} = \frac{AB}{AC}$
(ii) cosine A = $\frac{base}{hypotenuse} = \frac{AB}{AC}$	(ii) cosine C = $\frac{base}{hypotenuse} = \frac{BC}{AC}$
(iii) tangent A = $\frac{perpendicular}{base} = \frac{BC}{AB}$	(iii) tangent C = $\frac{perpendicular}{base} = \frac{AB}{BC}$
(iv) cosecant A = $\frac{hypotenuse}{perpendicular} = \frac{AC}{BC}$	(iv) cosecant C = $\frac{hypotenuse}{perpendicular} = \frac{AC}{AB}$
(v) secant A = $\frac{hypotenuse}{base} = \frac{AC}{AB}$	(v) secant C = $\frac{hypotenuse}{base} = \frac{AC}{BC}$
(v) cotangent A = $\frac{base}{perpendicular} = \frac{AB}{BC}$	(v) cotangent C = $\frac{base}{perpendicular} = \frac{BC}{AB}$

TRIGONOMETRIC IDENTITIES

An equation involving trigonometric ratio of angle(s) is called a trigonometric identity, if it is true for all values of the angles involved. These are:

tan θ = $\frac{sin\theta}{cos\theta}$

cot θ = $\frac{cos\theta}{sin\theta}$

- $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta \Rightarrow \cos^2\theta = 1 - \sin^2\theta$
- $\text{cosec}^2\theta - \cot^2\theta = 1 \Rightarrow \text{cosec}^2\theta = 1 + \cot^2\theta \Rightarrow \cot^2\theta = \text{cosec}^2\theta - 1$
- $\sec^2\theta - \tan^2\theta = 1 \Rightarrow \sec^2\theta = 1 + \tan^2\theta \Rightarrow \tan^2\theta = \sec^2\theta - 1$
- $\sin\theta \,\text{cosec}\,\theta = 1 \Rightarrow \cos\theta \sec\theta = 1 \Rightarrow \tan\theta \cot\theta = 1$

MOTIVATION…

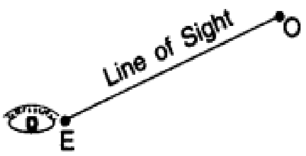
Value of t-ratios of specified angles:

∠A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
cosec A	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
cot A	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

CHAPTER 9

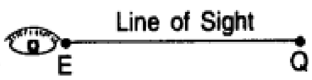
Line of Sight

When an observer looks from a point E (eye) at an object O then the straight line EO between the eye E and the object O is called the line of sight.



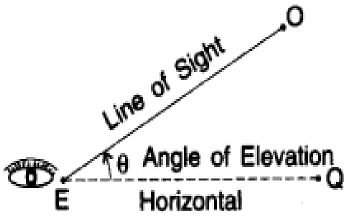
Horizontal

When an observer looks from a point E (eye) to another point Q which is horizontal to E, then the straight line, EQ between E and Q is called the horizontal line.



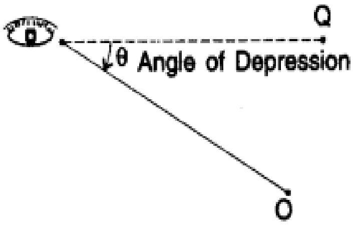
Angle of Elevation

When the eye is below the object, then the observer has to look up from the point E to the object O. The measure of this rotation (angle θ) from the horizontal line is called the angle of elevation.



Angle of Depression

When the eye is above the object, then the observer has to look down from the point E to the object. The horizontal line is now parallel to the ground. The measure of this rotation (angle θ) from the horizontal line is called the angle of depression.



- Choose a trigonometric ratio in such a way that it considers the known side and the side that you wish to calculate.
- The eye is always considered at ground level unless the problem specifically gives the height of the observer.

CHAPTER 10

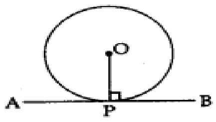
Centre: The fixed point is called the centre.

Radius: The constant distance from the centre is called the radius.

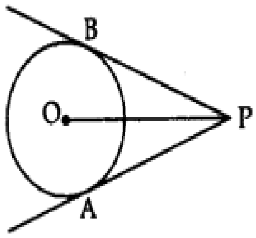
Chord: A line segment joining any two points on a circle is called a chord.

Diameter: A chord passing through the centre of the circle is called diameter. It is the longest chord.

Tangent: When a line meets the circle at one point or two coincidings The line is known as points, a tangent.
The tangent to a circle is perpendicular to the radius through the point of contact.
$\Rightarrow OP \perp AB$



The lengths of the two tangents from an external point to a circle are equal.
$\Rightarrow AP = PB$



Length of Tangent Segment

PB and PA are normally called the lengths of tangents from outside point P.

Properties of Tangent to Circle

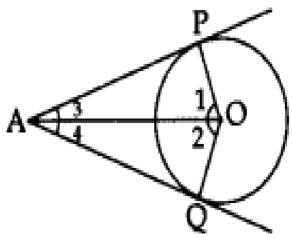
Theorem 1: Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Theorem 2: A line drawn through the end point of a radius and perpendicular to it, is the tangent to the circle.

Theorem 3: Prove that the lengths of tangents drawn from an external point to a circle are equal

Note: If two tangents are drawn to a circle from an external point, then:

- They subtend equal angles at the centre i.e., ∠1 = ∠2.
- They are equally inclined to the segment joining the centre to that point i.e., ∠3 = ∠4. ∠OAP = ∠OAQ



CHAPTER 11

Determining a Point Dividing a given Line Segment, Internally in the given Ratio M : N

Construction of a Tangent at a Point on a Circle to the Circle when its Centre is Known

Construction of a Tangent at a Point on a Circle to the Circle when its Centre is not Known

If the centre of the circle is not known, then we first find the centre of the circle by drawing two non-parallel chords of the circle. The point of intersection of perpendicular bisectors of these chords gives the centre of the circle. Then we can proceed as above.

Construction of a Tangents from an External Point to a Circle when its Centre is Known

Construction of a Tangents from an External Point to a Circle when its Centre is not Known

If the centre of the circle is not known, then we first find the centre of the circle by drawing two non-parallel chords of a circle. The point of intersection of perpendicular bisectors of the chords gives the centre of the circle. Then we can proceed as above.

Construction of a Triangle Similar to a given Triangle as per given Scale Factor $\frac{m}{n}$, m < n.

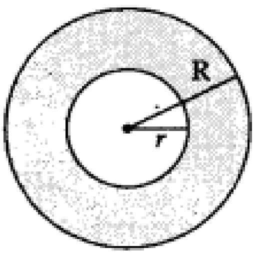
CHAPTER 12

Circumference of a circle = $2\pi r$

Area of a circle = πr^2 ...[where r is the radius of a circle]

Area of a semi-circle = $\frac{\pi r^2}{2}$

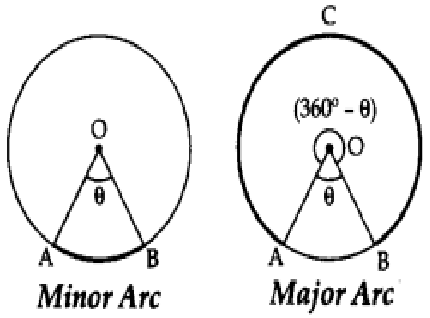
Area of a circular path or ring:



Let 'R' and 'r' he radii of two circles

Then area of shaded part = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2) = \pi(R + r)(R - r)$

Minor arc and Major Arc: An arc length is called a major arc if the arc length enclosed by the two radii is greater than a semi-circle.



If the arc subtends angle 'θ' at the centre, then the

Length of minor arc = $\frac{\theta}{360} \times 2\pi r = \frac{\theta}{180} \times \pi r$

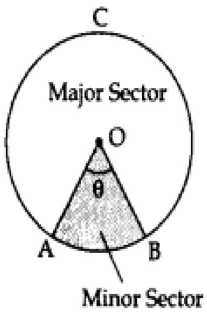
Length of major arc = $\left(\frac{360-\theta}{360}\right) \times 2\pi r$

Sector of a Circle and its Area

A region of a circle is enclosed by any two radii and the arc intercepted between two radii is called the sector of a circle.

(i) A sector is called a minor sector if the minor arc of the circle is part of its boundary.

$O\hat{A}B$ is minor sector.



Area of minor sector = $\frac{\theta}{360}\left(\pi r^2\right)$

Perimeter of minor sector = $2r + \frac{\theta}{360}(2\pi r)$

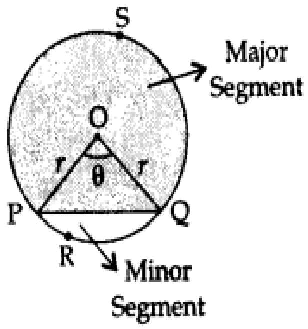
(ii) A sector is called a major sector if the major arc of the circle is part of its boundary.

$O\hat{A}CB$ is major sector

Area of major sector = $\left(\frac{360-\theta}{360}\right)\left(\pi r^2\right)$

Perimeter of major sector = $2r + \left(\frac{360-\theta}{360}\right)(2\pi r)$

Minor Segment: The region enclosed by an arc and a chord is called a segment of the circle. The region enclosed by the chord PQ & minor arc PRQ is called the minor segment.



Area of Minor segment = Area of the corresponding sector – Area of the corresponding triangle

$$= \left[\frac{\theta}{360}\pi r^2 - \frac{1}{2}r^2\sin\theta\right]$$

$$= \frac{1}{2}r^2\left[\frac{\theta}{180}\pi - \sin\theta\right] \text{ or } \frac{1}{2}r^2\left[\frac{\theta}{180}\pi - 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right]$$

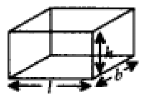
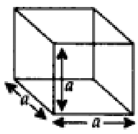
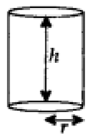
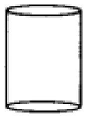
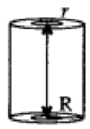
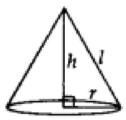
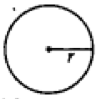
Major Segment: The region enclosed by the chord PQ & major arc PSQ is called the major segment.

Area of major segment = Area of a circle – Area of the minor segment

Area of major sector + Area of triangle

$$= \pi r^2 - \frac{\theta}{360}\pi r^2 + \frac{1}{2}r^2\sin\theta = r^2\left[\pi - \frac{\theta}{360}\pi + \frac{\sin\theta}{2}\right]$$

TABLE FOR SURFACE AREA AND VOLUME

Solid	Figures	Curved surface area (1)	Plane area (2)	Total area [1 + 2]	Volume	Remarks
Cuboid		Also known as lateral surface area $= 2(lh + bh)$	Area of: Top face $= lb$ Bottom face $= lb$ $\therefore lb + lb = 2lb$	$2(lb + bh + hl)$	$l.b.h$	l : length b : breadth h : height
Cube		Lateral surface area $= 4a^2$	Area of: Top face $= a^2$ Bottom face $= a^2$ $\therefore a^2 + a^2 = 2a^2$	$4a^2 + 2a^2 = 6a^2$	a^3	a : Side of cube
Right circular cylinder closed at top		Curved surface area $= 2\pi rh$	Area of: Top face $= \pi r^2$ Bottom face $= \pi r^2$ $\therefore \pi r^2 + \pi r^2 = 2\pi r^2$	$2\pi r^2 + 2\pi rh$ Or, $2\pi r(r + h)$	$\pi r^2 h$	r : radius h : height of cylinder
Right circular cylinder open at top		Curved surface area $= 2\pi rh$	Area of: Top face $= 0$ Bottom face $= \pi r^2$ $\therefore 0 + \pi r^2 = \pi r^2$	$2\pi rh + \pi r^2$ Or, $\pi r(2h + r)$	$\pi r^2 h$	r : radius h : height of cylinder
Hollow cylinder (Pipe)		$2\pi Rh$ • External surface area $= 2\pi Rh$ • Internal surface area $= 2\pi rh$	Area of: Top face $= \pi(R^2 - r^2)$ Bottom face $= \pi(R^2 - r^2)$	$2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$	$\pi R^2 h - \pi r^2 h$ (External Vol. – Internal Vol.)	R : Radius of outer base r : radius of inner base h = height
Cone		πrl	Area of: Bottom Face $= \pi r^2$	$\pi r^2 + \pi rl$ Or, $\pi r(r + l)$	$\frac{1}{3}\pi r^2 h$	h = height of cone r = radius of cone l = slant height $= \sqrt{h^2 + r^2}$
Sphere		$4\pi r^2$	None	$4\pi r^2$	$\frac{4}{3}\pi r^3$	r : radius of sphere
Hemisphere		$2\pi r^2$	πr^2	$3\pi r^2$	$\frac{2}{3}\pi r^3$	r : radius of hemisphere
Spherical shell		$4\pi R^2$ (Outer) $4\pi r^2$ (Inner)	None	$4\pi R^2 + 4\pi r^2$	$\frac{4}{3}\pi \cdot (R^3 - r^3)$	R:Radius of outer shell r:Radius of inner shell

CHAPTER 14

MEAN (AVERAGE): Mean [Ungrouped Data] – Mean of n observations, $x_1, x_2, x_3 \ldots x_n$, is

$$\overline{X} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \frac{1}{n}\Sigma x \qquad \therefore \qquad \overline{X} = \frac{\Sigma x}{n}$$

MEAN [Grouped Data]: The mean for grouped data can be found by the following three methods:

(i) Direct Mean Method:

$$\overline{X} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

Class Mark = $\frac{Upper \quad Class \quad Limit+Lower \quad Class \quad Limit}{2}$

Note: Frequency of a class is centred at its mid-point called class mark.

(ii) Assumed Mean Method: In this, an arbitrary mean 'a' is chosen which is called, 'assumed mean', somewhere in the middle of all the values of x.

$$\overline{X} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

MEDIAN: Median is a measure of central tendency which gives the value of the middle-most observation in the data.

(i) Ungrouped data: *If n is odd → Median =* $\left(\frac{n+1}{2}\right)^{th}$ *observation*

If n is even → Median = $\frac{\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2}+1\right)^{th} \text{observation}}{2}$

Remember! For ungrouped data, first arrange the observations in ascending order or descending order.

(ii) Median (Grouped Data): Median = $l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$

...where[l = Lower limit of median class; n = Number of observations; f = Frequency of median class; c.f. = Cumulative frequency of preceding class; h = Class size]

Mode:

(i) Ungrouped Data: The value of the observation having maximum frequency is the mode.

(ii) Grouped Data:

$$Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

...where[l = Lower limit of modal class; f_1 = Frequency of modal class; f_0 = Frequency of the class preceding the modal class; f_2 = Frequency of the class succeeding the modal class; h = Size of class interval. c.f. = Cumulative frequency of preceding class; h = Class size]

Mode = 3 Median – 2 Mean

Median = $\frac{Mode+2Mean}{3}$

Mean = $\frac{3Median-Mode}{2}$

CHAPTER 15

Probability: It is the numerical measurement of the degree of certainty.

- Theoretical probability associated with an event E is defined as "If there are 'n' elementary events associated with a random experiment and m of these are favourable to the event E then the probability of occurrence of an event is defined by P(E) as the ratio $\frac{m}{n}$ ".

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}. \quad \text{Thus, } P(E) = \frac{m}{n}$$

- If P(E) = 1, then it is called a 'Certain Event'.
- If P(E) = 0, then it is called an 'Impossible Event'.
- The probability of an event E is a number P(E) such that: $0 \leq P(E) \leq 1$
- An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.
- For any event E, P(E) + P($\bar{E}$) = 1, where $\bar{E}$ stands for 'not E'. E and $\bar{E}$ are called complementary events.
- Favourable outcomes are those outcomes in the sample space that are favourable to the occurrence of an event.

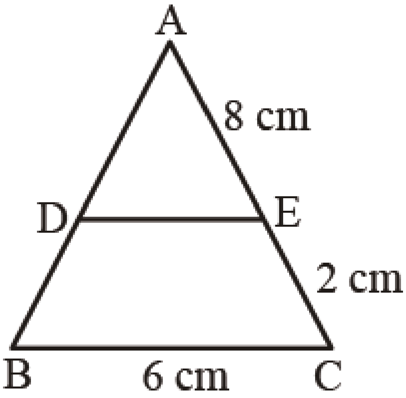
Sample Space

A collection of all possible outcomes of an experiment is known as sample space. It is denoted by 'S' and represented in curly brackets.

One mark questions with sample

1

In the given figure if DE || BC, AE = 8 cm, EC = 2 cm and BC = 6 cm, then find DE.



In ΔADE and ΔABC,

$\angle DAE = \angle BAC$ [Common]

$\angle ADE = \angle ABC$ [Corresponding angles]

By AA axiom of similarity

$\Delta ADE \sim \Delta ABC$

$$\therefore \frac{AE}{AC} = \frac{DE}{BC}$$

$$\Rightarrow \frac{8}{8+2} = \frac{DE}{6}$$

$$\Rightarrow DE = 4.8 \text{ cm}$$

2

If cosec $\theta = \frac{5}{4}$, find the value of cot θ.

$$\cot^2\theta = \text{cosec}^2\theta - 1$$

$$= \left(\frac{5}{4}\right)^2 - 1 = \frac{25}{16} - 1 = \frac{25-16}{16} = \frac{9}{16}$$

$$\Rightarrow \cot^2\theta = \frac{9}{16}$$

$$\cot\theta = \frac{3}{4}$$

3 *If $xy = 180$ and HCF $(x, y) = 3$, then find the LCM(x, y).*

Product of numbers = LCM(x, y) × HCF(x, y)

$\Rightarrow$ LCM $(x, y) \times (3) = 180$

$\Rightarrow$ LCM $(x, y) = 60$

4 **Following table shows sale of shoes in a store during one month :**

Size of shoe	3	4	5	6	7	8
Number of pairs sold	4	18	25	12	5	1

Maximum number of pairs sold = 25 (size 5)

Model size of shoes = 5

5 *The decimal representation of $\frac{14587}{2^1 \times 5^4}$ will terminate after how many decimal places?*

Here $\frac{14587}{2 \times 5^4} = \frac{14587}{2 \times 625} = \frac{14587}{1250} = 11.6696$

$\therefore$ Four decimal places

6 *If the sum of the zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then find the value of k.*

If α and β are sum of the zeroes of $3x^2 - kx + 6$ then

$$\alpha + \beta = \frac{-(-k)}{3} = \frac{k}{3}$$

$$\Rightarrow \quad 3 = \frac{k}{3}$$

$$\Rightarrow \quad k = 9$$

7 *For what value of k, the pair of linear equations 3x + y = 3 and 6x + ky = 8 does not have a solution.*

The given pair of linear equations does not have a solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{3}{6} = \frac{1}{k} \neq \frac{3}{8} \quad \Rightarrow \quad \frac{3}{6} = \frac{1}{k} \quad \Rightarrow \quad k = 2$$

8 *If 3 chairs and 1 table costs ₹ 1500 and 6 chairs and 1 table costs ₹ 2400. Form linear equations to represent this situation.* (1)

Let the cost of 1 chair = ₹ x

And the cost of 1 table = ₹ y

Now according to the question,

$$3x + y = 1500 \quad \text{and} \quad 6x + y = 2400$$

9 **Which term of the AP 27, 24, 21, ... is zero?**

Let nth term of the given A.P. is zero then

$$a_n = a + (n - 1)d$$

$$\Rightarrow \quad 0 = 27 + (n - 1)(-3)$$

$$\Rightarrow \quad 0 = 27 - 3n + 3$$

$$\Rightarrow \quad 30 = 3n \quad \Rightarrow \quad n = 10$$

10 **n^{th} term of an AP is given by**

n^{th} term of an AP is given by

$$a_n = a + (n - 1)d$$

$$\Rightarrow \quad 4 = a + (7 - 1) \times (-4)$$

$$\Rightarrow \quad 4 = a + 6 \times (-4) \quad \Rightarrow \quad a = -28$$

11 *For what values of k, the equation $9x^2 + 6kx + 4 = 0$ has equal roots?*

The given quadratic equation $9x^2 + 6kx + 4 = 0$ has equal roots then $b^2 - 4ac = 0$

$$(6k)^2 - 4 \times 9 \times 4 = 0$$

$\Rightarrow$ $36k^2 = 144$ $\Rightarrow$ $k^2 = 4$ $\Rightarrow$ $k = \pm 2$

12 *Find the roots of the equation $x^2 + 7x + 10 = 0$*

The given equation is

$x^2 + 7x + 10 = 0$ $\Rightarrow$ $x^2 + 5x + 2x + 10 = 0$

$\Rightarrow$ $x(x + 5) + 2(x + 5) = 0$ $\Rightarrow$ $(x + 5)(x + 2) = 0$

$\Rightarrow$ $x = -5,\ x = -2$

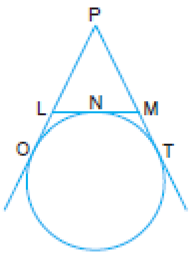
13 ***For what value(s) of 'a' quadratic equation $3ax^2 - 6x + 1 = 0$ has no real roots?***

The given equation $3ax^2 - 6x + 1 = 0$ has no real roots *i.e.* $b^2 - 4ac < 0$

$\Rightarrow$ $(-6)^2 - 4(3a)(1) < 0$ $\Rightarrow$ $36 - 12a < 0$

$\Rightarrow$ $12a > 36$ $\Rightarrow$ $a > 3$

14 *If PQ = 28 cm, then find the perimeter of ΔPLM.*



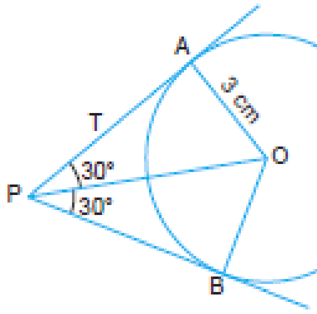
If two tangents inclined at 60° are drawn to a circle of radius 3 cm then find the length of each tangent. (1)

$$PQ = PT$$

$PL + LQ = PM + MT$ $\Rightarrow$ $PL + LN = PM + MN$

$$\begin{aligned}\text{Perimeter } (\Delta PLM) &= PL + LM + PM \\ &= PL + LN + MN + PM \\ &= 2(PL + LN) = 2(PL + LQ) \\ &= 2 \times 28 = 56 \text{ cm}\end{aligned}$$

15 *PQ is a tangent to a circle with centre O at point P. If ΔOPQ is an isosceles triangle, then find $\angle OQP$.*



In Δ OPQ

$$\angle P + \angle Q + \angle O = 180°$$

$\Rightarrow$ $2\angle Q + \angle P = 180°$ $\quad$ ($\because$ Δ OPQ is an isosceles triangle)

$\Rightarrow$ $2\angle Q + 90° = 180°$

$\Rightarrow$ $2\angle Q = 90°$ $\quad \Rightarrow \quad$ $\angle Q = 45°$

16 *If two tangents inclined at 60° are drawn to a circle of radius 3 cm then find the length of each tangent.* (1)

According to given condition in question

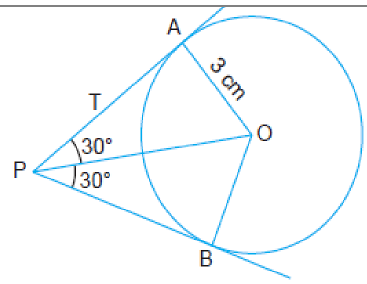
PA and PB are two tangents drawn to a circle then

In Δ PAO

$$\tan 30° = AO/PA$$

$\Rightarrow$ $1/\sqrt{3} = 3/PA$

$\Rightarrow$ $PA = 3\sqrt{3}$ cm $= PB$



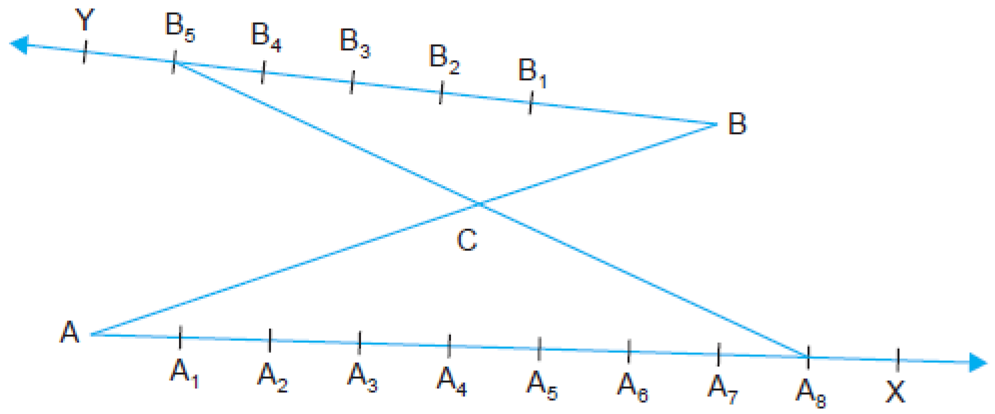
17 *In the ΔABC, D and E are points on side AB and AC respectively such that $DE \parallel BC$. If AE = 2 cm, AD = 3 cm and BD = 4.5 cm then find CE.* (1)

According to mid-point theorem

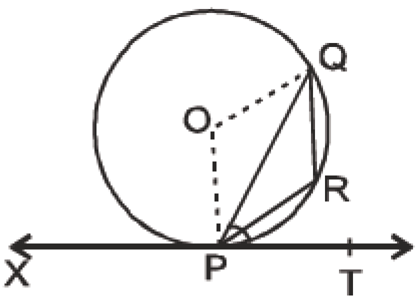
$$\frac{AD}{BD} = \frac{AE}{CE}$$

$\therefore$ $$\frac{3}{4.5} = \frac{2}{CE}$$

$\Rightarrow$ $CE = \dfrac{4.5 \times 2}{3} = 1.5 \times 2$ $\quad \Rightarrow \quad$ CE = 3 cm

18	*In the figure, if B_1, B_2, B_3 ..... and A_1, A_2, A_3 ..... have been marked at equal distances. In what ratio C divides AB?* (1) 
	8:5
19	*$\sin A + \cos B = 1$, $A = 30°$ and B is an acute angle, then find the value of B.*
	Given $\sin 30° + \cos B = 1$ $\Rightarrow \quad \frac{1}{2} + \cos B = 1 \quad \Rightarrow \quad \cos B = 1 - \frac{1}{2} = \frac{1}{2}$ $\Rightarrow \quad \cos B = \cos 60° \quad \Rightarrow \quad B = 60°$
20	*If $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$, then find $x + y$.*
	$x + y = 2\sin^2\theta + 2\cos^2\theta + 1$ $= 2(\sin^2\theta + \cos^2\theta) + 1 \quad [\because \ (\sin^2\theta + \cos^2\theta = 1)]$ $= 3$
21	*In a circle of diameter 42 cm, if an arc subtends an angle at 60° at the centre where $\pi = 22/7$, then what will be the length of arc.* (1)
	Length of arc $= \theta/360° \, (2\pi r) = 60/360 \, (2 \times 22/7 \times 21) = 22$ cm
22	*12 solid spheres of the same radii are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. Find the diameter of the each sphere.* (1)
	According to question, $\pi R^2 H = 12 \times 4/3 \pi r^3$ where r is radius of sphere $1 \times 1 \times 16 = 4/3 \times r^3 \times 12$ $\Rightarrow \quad r^3 = 1 \quad \Rightarrow \quad r = 1$
23	*Find the probability of getting a doublet in a throw of a pair of dice.*

	Probability of getting a doublet = $\frac{1}{6}$
24	***Find the probability of getting a black queen when a card is drawn at random from a well-shuffled pack of 52 cards.***
	Probability of getting a black queen = $\frac{2}{52} = \frac{1}{26}$
25	**Find the values of k for which the quadratic equation $9x^2 - 3kx + k = 0$ has equal roots.**
	For equal roots, D = 0 $\therefore\ b^2 - 4ac = 0$ $\Rightarrow (-3k)^2 - 4 \times 9 \times k = 0$ $\Rightarrow 9k^2 - 36k = 0 \quad \Rightarrow \quad 9k(k-4) = 0$ $\Rightarrow$ Either $k = 0$ or $k - 4 = 0$ *i.e.* $k = 0$ or 4
26	**If $\sqrt{3}\sin\theta = \cos\theta$, find the value of $\frac{3\cos^2\theta + 2\cos\theta}{3\cos\theta + 2}$.**
	$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$ or $\tan\theta = \frac{1}{\sqrt{3}}$ $\Rightarrow \tan\theta = \tan 30^\circ \Rightarrow \theta = 30^\circ$ Now, $\frac{3\cos^2\theta + 2\cos\theta}{3\cos\theta + 2} = \frac{\cos\theta(3\cos\theta + 2)}{(3\cos\theta + 2)}$ $= \cos\theta = \cos 30^\circ = \frac{\sqrt{3}}{2}$
27	**If the quadratic equation $px^2 - 2\sqrt{5}px + 15 = 0$ has two equal roots, then find the value of p.**

	For equal roots, D = 0 $\Rightarrow b^2 - 4ac = 0$ $\Rightarrow (-2\sqrt{5})^2 - 4 \times p \times 15 = 0$ $\Rightarrow 20 - 60p = 0$ $\Rightarrow p = \dfrac{1}{3}$
28	**Two different dice are tossed together. Find the probability that the product of the two numbers on the top of the dice is 6.**
	Let A be the event such that A = Product of the numbers on the top of the dice is 6. $\therefore$ Favourable outcomes (1, 6), (2, 3), (3, 2), (6, 1) $\therefore$ No. of favourable out comes = 4 $\therefore$ Required probability = P (A) $= \dfrac{4}{36} = \dfrac{1}{9}$
29	**In Figure given below PQ is a chord of a circle with centre O and PT is a tangent. If $\angle QPT = 60^\circ$, find $\angle PRQ$.** 

$\angle XPQ + \angle QPT = 180^\circ$ (linear pair)

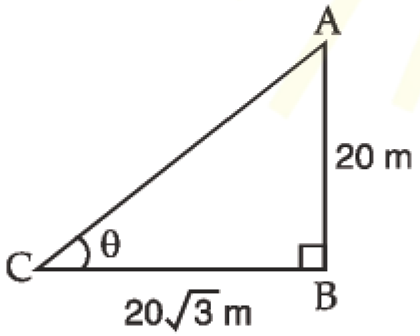
$\angle XPQ + 60^\circ = 180^\circ$

$\angle XPQ = 120^\circ$

Now $\angle XPQ = \angle PRQ$ [Angle made by a chord with a tangent is equal to angle subtended by the chord in alternate segment]

$\angle PRQ = 120^\circ$

30 **In Figure given below a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long. Find the Sun's altitude.**



Let Sun's altitude = $\theta = \angle ACB$

In right angled ΔABC, we have

$$\tan\theta = \frac{AB}{BC}$$

$$\tan\theta = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^\circ \Rightarrow \theta = 30^\circ$$

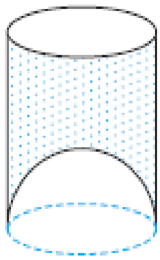
Try these…practice(1 or 2 marks)

Solve and get it checked it from subject teacher

1 *The HCF of two numbers a and b is 5 and their LCM is 200. Find the product ab.*

2	Find the value of k for which $x = 2$ is a solution of the equation $kx^2 + 2x - 3 = 0$.
3	If in an A. P., $a = 15$, $d = -3$ and $a_n = 0$, then find the value of n.
4	If $\sin x + \cos y = 1$; $x = 30°$ and y is an acute angle, find the value of y.
5	The area of two similar triangles are 25 sq. cm and 121 sq. cm. Find the ratio of their corresponding sides.
6	Find the value of 'a' so that the point (3, a) lies on the line represented by $2x - 3y = 5$.
7	If S_n, the sum of the first n terms of an A.P. is given by $S_n = 2n^2 + n$, then find its n^{th} term.
8	The mid-point of the line segment joining A(2a, 4) and B(-2, 3b) is $(1, 2a + 1)$. Find the values of a and b.
9	A child has a die whose 6 faces show the letters given below: A B C A A B The die is thrown once. What is the probability of getting (i) A (ii) B ?
10	Find the HCF of 612 and 1314 using prime factorisation.
11	Cards marked with numbers 5 to 50 (one number on one card) are placed in a box and mixed thoroughly. One card is drawn at random from the box. Find the probability that the number on the card taken out is (i) a prime number less than 10, (ii) a number which is a perfect square.

12	*For what value of k, does the system of linear equations* $$2x + 3y = 7$$ $$(k - 1)x + (k + 2)y = 3k$$ *have an infinite number of solutions ?*
13	Prove that $\sqrt{5}$ is an irrational number.
14	Find all the zeroes of the polynomial $x^4 + x^3 - 14x^2 - 2x + 24$, if two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$. Let $p(x) = x^4 + x^3 - 14x^2 - 2x + 24$
15	Point P divides the line segment joining the points A(2, 1) and B(5, -8) such that $\frac{AP}{AB} = \frac{1}{3}$. If P lies on the line $2x - y + k = 0$, find the value of k.
16	Prove that: $\frac{\tan\theta}{1 - \tan\theta} - \frac{\cot\theta}{1 - \cot\theta} = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$
17	A part of monthly hostel charges in a college hostel are fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 25 days, he has to pay ₹4,500, whereas a student B who takes food for 30 days, has to pay ₹5,200. Find the fixed charges per month and the cost of food per day.
18	In $\triangle ABC$, $\angle B = 90°$ and D is the mid-point of BC. Prove that $AC^2 = AD^2 + 3CD^2$.
19	Prove that the parallelogram circumscribing a circle is a rhombus.

20	In Figure, three sectors of a circle of radius 7 cm, making angles of 60°, 80° and 40° at the centre are shaded. Find the area of the shaded region.
21	If $\tan 2A = \cot (A - 18°)$, where 2A is an acute angle, find the value of A.
22	*A juice seller was serving his customers using glasses as shown in Figure. The inner diameter of the cylindrical glass was 5 cm but bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent and actual capacity of the glass. (Use $\pi = 3.14$)* 
	2 MARKS, 3 MARKS & 5 MARKS WITH
	SURE SHOT…
1	**Find whether the following pair of linear equations is consistent or inconsistent :** $3x + 2y = 8$ $6x - 4y = 9$ **[2]**
	Here, $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{2}{-4} = \frac{-1}{2}$. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, which will give a unique solution.
2	**Explain why $(17 \times 5 \times 11 \times 3 \times 2 + 2 \times 11)$ is a composite number ? [2]**

$17 \times 5 \times 11 \times 3 \times 2 + 2 \times 11$

$= 17 \times 5 \times 3 \times 22 + 22$

$= 22\,(17 \times 5 + 3 \times 1)$

$= 22\,(255 + 1) = 2 \times 11 \times 256$

Given expression is divisible by 2, 11 and 256, which means it has more than 2 prime factors.

$(17 \times 5 \times 11 \times 3 \times 2 + 2 \times 11)$ is a composite number. **Ans.**

3 **Prove the following identity :**

$$\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} = 1 - \sin\theta \cdot \cos\theta$$

$$\text{L.H.S.} = \frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta}$$

$$= \frac{(\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta \cdot \cos\theta)}{(\sin\theta + \cos\theta)}$$

$= 1 - \sin\theta \cdot \cos\theta = \text{R.H.S.}$ [$\because$

4 **Show that the mode of the series obtained by combining the two series S_1 and S_2 given below is different from that of S_1 and S_2 taken separately :**

S_1 : 3, 5, 8, 8, 9, 12, 13, 9, 9

S_2 : 7, 4, 7, 8, 7, 8, 13 **[2]**

Mode of S_1 series = 9 [_ observation 9 repeated max. no. of times *i.e.* 3 times]

Mode of S_2 series = 7 [_ observation 7 repeated max no. of times *i.e.* 3 times]

After combining S_1 and S_2, the new series will be ; 3, 5, 8, 8, 9, 12, 13, 9, 9, 7, 4, 7, 8, 7, 8, 13.

Mode of combined series = 8 (maximum times *i.e.* 4 times)

Mode of (S_1, S_2) is different from mode of S_1 and mode of S_2 separately. **Hence Proved.**

5 **Solve by elimination :**

$\mathbf{3x - y = 7}$

$\mathbf{2x + 5y + 1 = 0}$

	Given equations are : $3x - y = 7$ ...(*i*) $2x + 5y = -1$ ...(*ii*) Multiplying equation (*i*) by 5 and solving it with equation (*ii*), we get $2x + 5y = -1$ $15x - 5y = 35$ $17x = 34$ (on Adding) $x = \frac{34}{17} = 2$ Putting the value of x in (*i*), we have $3(2) - y = 7$ $\Rightarrow 6 - y = 7 \Rightarrow -y = 7 - 6 \quad \Rightarrow y = -1$ $\therefore\ x = 2, y = -1$ **Ans.**
6	**If $\sec\theta + \tan\theta = p$, prove that $\sin\theta = \frac{p^2 - 1}{p^2 + 1}$**
	R.H.S. $= \frac{p^2 - 1}{p^2 + 1}$ $= \frac{(\sec\theta + \tan\theta)^2 - 1}{(\sec\theta + \tan\theta)^2 + 1} = \frac{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta - 1}{\sec^2\theta + \tan^2 + 2\sec\theta\tan\theta + 1}$ [By $(a + b)^2 = a^2 + b^2 + 2ab$] $= \frac{(\sec^2\theta - 1) + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + (1 + \tan^2\theta) + 2\sec\theta\tan\theta}$ $= \frac{\tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + \sec^2\theta + 2\sec\theta\tan\theta}$ $\left[\begin{array}{l}\because \sec^2\theta - 1 = \tan^2\theta \\ \text{and } \sec^2\theta = 1 + \tan^2\theta\end{array}\right]$ $= \frac{2\tan^2\theta + 2\sec\theta\tan\theta}{2\sec^2\theta + 2\sec\theta\tan\theta} = \frac{2\tan\theta(\tan\theta + \sec\theta)}{2\sec\theta(\sec\theta + \tan\theta)} = \frac{\tan\theta}{\sec\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$ $= \sin\theta =$ L.H.S.
7	**The average score of boys in the examination of a school is 71 and that of the girls is 73. The average score of the school in the examination is 71.8. Find the ratio of number of boys in the number of girls who appeared in the examination. [3]**

Let the number of boys = n_1

and number of girls = n_2

Average boys' score = 71 $= \bar{X}_1$ (Let)

Average girls' score = 73 $= \bar{X}_2$ (Let)

Combined mean $= \dfrac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$

$$71.8 = \frac{n_1(71) + n_2(73)}{n_1 + n_2} \quad \Rightarrow \quad 71n_1 + 73n_2 = 71.8n_1 + 71.8n_2$$

$$71n_1 - 71.8n_1 = 71.8n_2 - 73n_2 \;\Rightarrow\; -0.8n_1 = -1.2n_2$$

$$\frac{n_1}{n_2} = \frac{1.2}{0.8} \Rightarrow \frac{n_1}{n_2} = \frac{3}{2} \quad \Rightarrow n_1 : n_2 = 3 : 2$$

No. of boys : No. of girls = 3 : 2. **Ans.**

8 **Prove that :**

$$(1 + \cot A + \tan A) \cdot (\sin A - \cos A) = \frac{\sec^3 A - \mathrm{cosec}^3 A}{\sec^2 A \cdot \mathrm{cosec}^2 A}$$

L.H.S. = (1 + cot A + tan A) (sin A – cos A)

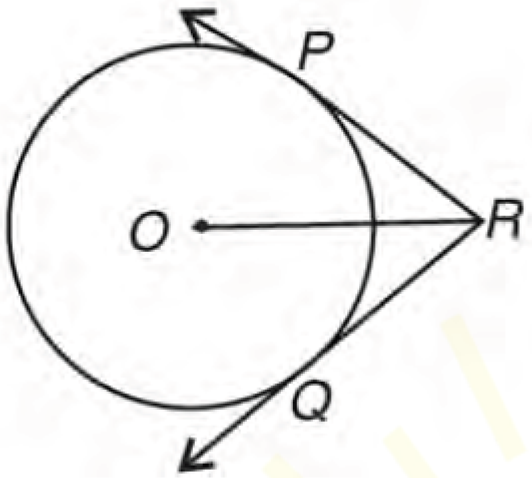
$$= \left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)(\sin A - \cos A) = \left(\frac{\sin A\cos A + \cos^2 A + \sin^2 A}{\sin A . \cos A}\right)(\sin A - \cos A)$$

$$= \frac{\sin^3 A - \cos^3 A}{\sin A . \cos A} \qquad [\text{Using } a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$$

$$= \frac{\dfrac{\sin^3 A}{\sin^3 A . \cos^3 A} - \dfrac{\cos^3 A}{\sin^3 A . \cos^3 A}}{\dfrac{\sin A \cos A}{\sin^3 A . \cos^3 A}} \qquad [\text{Dividing Num. and Deno. by } \sin^3 A \,.\, \cos^3 A]$$

$$= \frac{\sec^3 A - \mathrm{cosec}^3 A}{\sec^2 A . \mathrm{cosec}^2 A} = \text{R.H.S.}$$

Hence Proved.

9	**In Figure given below two tangents RQ and RP are drawn from an external point R to the circle with centre O. If ∠PRQ = 120°, then prove that OR = PR + RQ.** 

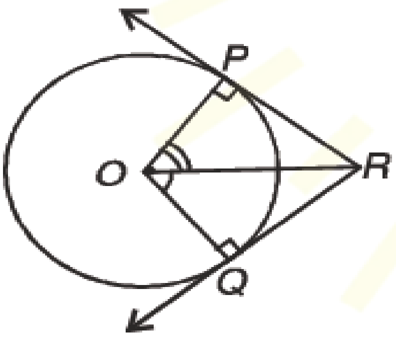
$\therefore \ \angle PRO = \angle QRO = 60^\circ$

In right ΔOPR $\qquad (\because OP \perp PR)$

$$\frac{PR}{OR} = \cos 60^\circ = \frac{1}{2}$$

$\Rightarrow OR = 2PR \qquad \ldots(i)$

Similarly $\dfrac{QR}{OR} = \dfrac{1}{2}$



$\Rightarrow OR = 2QR \qquad \ldots(ii)$

$(i) + (ii)$, we get

$2OR = 2PR + 2QR$

Thus, $OR = PR + RQ$

10 **The 14th term of an AP is twice its 8th term. If its 6th term is – 8, then find the sum of its first 20 terms.**

Let 1st term of AP = a and
common difference = d
According to given condition, we have

$a_{14} = 2a_8$

$a + 13d = 2(a + 7d) \Rightarrow a = -d$

$[\because a_n = a + (n-1)d]$

Also $a_6 = 8 \Rightarrow a + 5d = -8$

$-d + 5d = -8 \Rightarrow 4d = -8 \Rightarrow d = -2$

$a = 2$

We know that $S_n = \dfrac{n}{2}\{2a + (n-1)d\}$

$S_{20} = \dfrac{20}{2}(2 \times 2 + 19 \times (-2))$

$= 10 \times (-34) = -340$

Case study questions

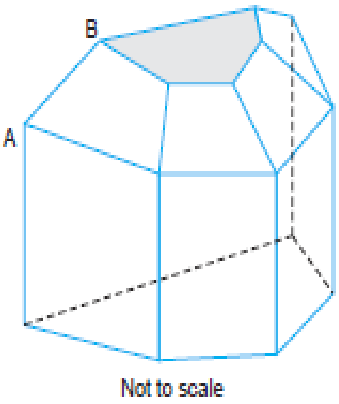
1

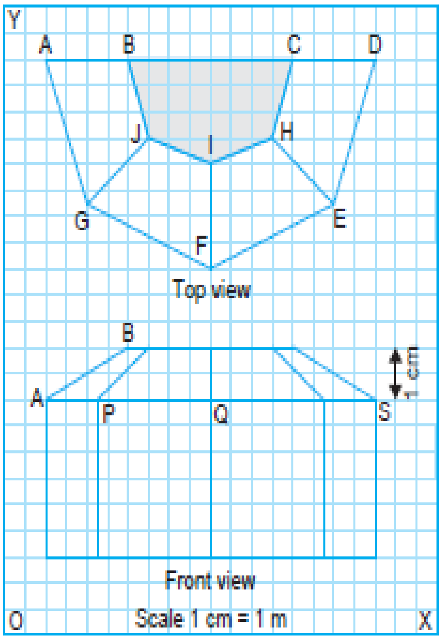
17. *Case Study based-1*

SUN ROOM

The diagrams show the plans for a sun room. It will be built onto the wall of a house. The four walls of the sunroom are square clear glass panels. The roof is made using

- *Four clear glass panels, trapezium in shape, all the same size*
- *One tinted glass panel, half a regular octagon in shape*





(a) Refer to Top View

Find the mid-point of the segment joining the points J(6, 17) and I (9, 16). **(1)**

(i) (33/2, 15/2) (ii) (3/2, 1/2) (iii) (15/2, 33/2) (iv) (1/2, 3/2)

(b) Refer to Top View

The distance of the point P from the y-axis is **(1)**

(i) 4 (ii) 15 (iii) 19 (iv) 25

(c) Refer to Front View

The distance between the points A and S is **(1)**

(i) 4 (ii) 8 (iii) 16 (iv) 20

(d) Refer to Front View

Find the co-ordinates of the point which divides the line segment joining the points A and B in the ratio 1 : 3 internally. **(1)**

(i) (8.5, 2.0) (ii) (2.0, 9.5) (iii) (3.0, 7.5) (iv) (2.0, 8.5)

(e) Refer to Front View

If a point (x, y) is equidistant from the Q(9, 8) and S(17, 98) then **(1)**

(i) $x + y = 13$ (ii) $x - y = 0$ (iii) $y - 13 = 0$ (iv) $x - y = 13$

$(a)\,(iii)\left(\frac{15}{2}, \frac{33}{2}\right)$ $(b)\,(i)\,4$ $(c)\,(iii)\,16$ $(d)\,(iv)\,(2.0, 8.5)$ $(e)\,(ii)\,x - 13 = 0$

2

18. *Case Study Based-2*

SCALE FACTOR AND SIMILARITY

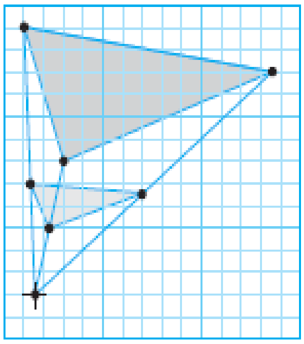
SCALE FACTOR

A scale drawing of an object is the same shape as the object but a different size.

The scale of a drawing is a comparison of the length used on a drawing to the length it represents. The scale is written as a ratio.

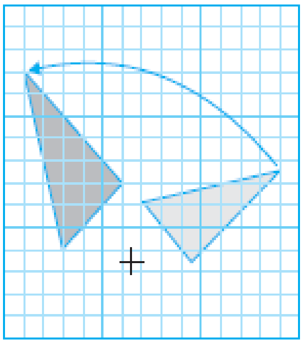
SIMILAR FIGURES

The ratio of two corresponding sides in similar figures is called the scale factor.

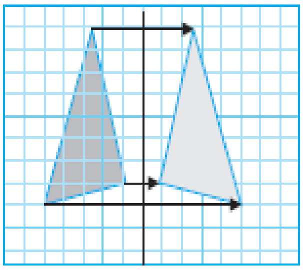


$$\text{Scale factor} = \frac{\text{length in image}}{\text{corresponding length in object}}$$

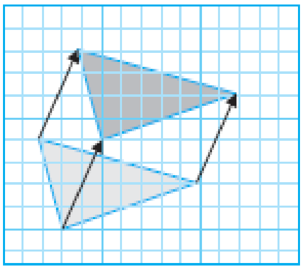
If one shape can become another using Resizing then the shapes are similar.



Rotation or Turn



Reflection or Flip



Translation or Slide

Hence the two shapes are similar when one can become the other after a resize, flip, slide or turn.

(a) A model of a boat is made on the scale of 1 : 4. The model is 120 cm long. The full size of the boat has a width of 60 cm. What is the width of the scale model? (1)

(i) 20 cm (ii) 25 cm (iii) 15 cm (iv) 240 cm

(b) What will effect the similarity of any two polygons? (1)

(i) They are flipped horizontally (ii) They are dilated by a scale factor

(iii) They are translated down (iv) They are not the mirror image of one another.

(c) *If two similar triangles have a scale factor of a : b. Which statement regarding the two triangles is true?* (1)

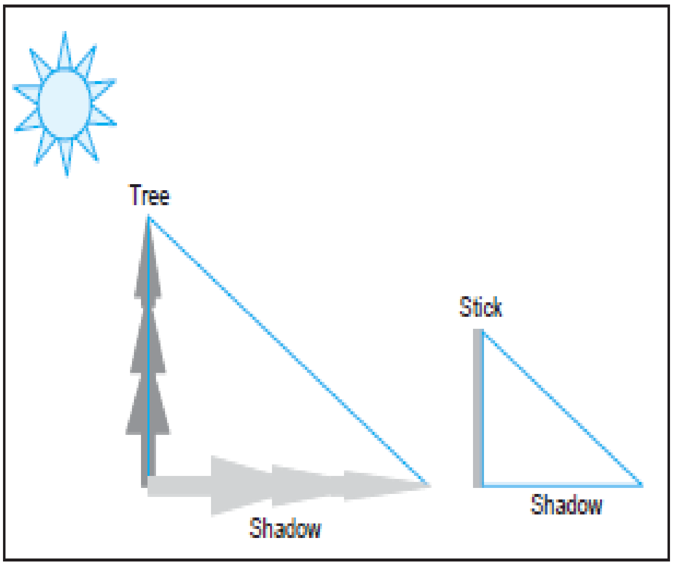
(i) *The ratio of their perimeters is 3a : b*

(ii) *Their altitudes have a ratio a : b*

(iii) *Their medians have a ratio* $\frac{a}{2} : b$

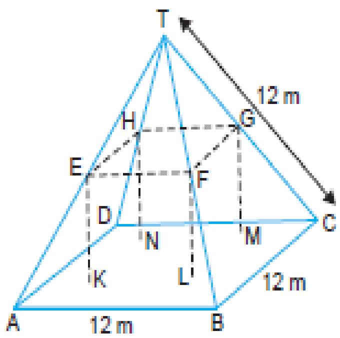
(iv) *Their angle bisectors have a ratio* $a^2 : b^2$

(d) *The shadow of a stick 5 m long is 2 m. At the same time the shadow of a tree 12.5 m high is* (1)



(i) *3 m* (ii) *3.5 m* (iii) *4.5 m* (iv) *5 m*

(e) *Below you see a student's mathematical model of a farmhouse roof with measurements. The attic floor, ABCD in the model, is a square. The beams that support the roof are the edges of a rectangular prism, EFGHKLMN. E is the middle of AT, F is the middle of BT, G is the middle of CT, and H is the middle of DT. All the edges of the pyramid in the model have length of 12 m. What is the length of EF, where EF is one of the horizontal edges of the block?* (1)



(i) *24 m* (ii) *3 m* (iii) *6 m* (iv) *10 m*

(a) (iii) 15 cm

(b) (iv) They are not the mirror image of one another

(c) (ii) Their altitudes have a ratio $a : b$

(d) (iv) 5 m

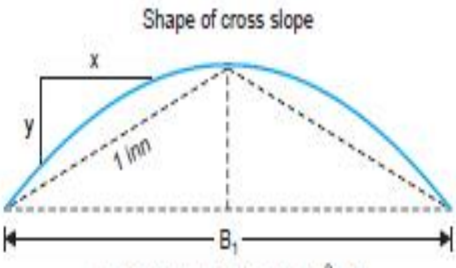
(e) (iii) 6 m

3

Case Study Based-3

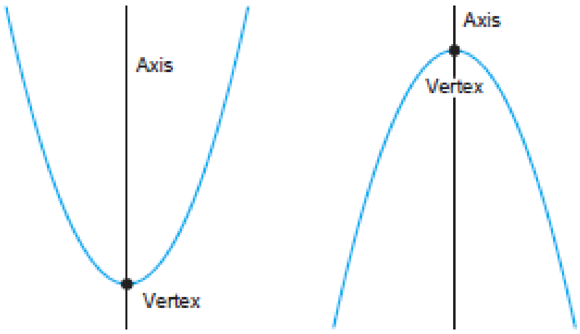
Applications of Parabolas-Highway Overpasses/Underpasses

A highway underpass is parabolic in shape.



(a) Parabolic Camber $y = 2x^2/nw$

Parabola: A parabola is the graph that results from $p(x) = ax^2 + bx + c$. Parabolas are symmetric about a vertical line known as the Axis of Symmetry. The Axis of Symmetry runs through the maximum or minimum point of the parabola which is called the Vertex.



(a) If the highway overpass is represented by $x^2 - 2x - 8$. Then its zeroes are **(1)**

(i) (2, – 4) *(ii) (4, – 2)* *(iii) (– 2, – 2)* *(iv) (– 4, – 4)*

(b) The highway overpass is represented graphically.

Zeroes of a polynomial can be expressed graphically. Number of zeroes of polynomial is equal to number of points where the graph of polynomial **(1)**

(i) Intersects x-axis *(ii) Intersects y-axis*

(iii) Intersects y-axis or x-axis *(iv) None of the above*

(c) Graph of a quadratic polynomial is a **(1)**

(i) straight line *(ii) circle* *(iii) parabola* *(iv) ellipse*

(d) The representation of Highway Underpass whose one zero is 6 and sum of the zeroes is 0, is

(i) $x^2 - 6x + 2$ *(ii) $x^2 - 36$* *(iii) $x^2 - 6$* *(iv) $x^2 - 3$* **(1)**

(e) The number of zeroes that polynomial $f(x) = (x - 2)^2 + 4$ can have is **(1)**

(i) 1 *(ii) 2* *(iii) 0* *(iv) 3*

(a) (ii) (4, – 2) (b) (i) intersects x-axis (c) (iii) parabola

(d) (ii) $x^2 - 36$ (e) (iii) 0

4

Case Study Based-4

100 m Race

A stopwatch was used to find the time that it took a group of students to run 100 m.

Time (in sec.)	0–20	20–40	40–60	60–80	80–100
No. of students	8	10	13	6	3

(a) Estimate the mean time taken by a student to finish the race. (1)

(i) 54 (ii) 63 (iii) 43 (iv) 50

(b) What will be the upper limit of the modal class? (1)

(i) 20 (ii) 40 (iii) 60 (iv) 80

(c) The construction of cummulative frequency table is useful in determining the (1)

(i) Mean (ii) Median (iii) Mode (iv) All of the above

(d) The sum of lower limits of median class and modal class is (1)

(i) 60 (ii) 100 (iii) 80 (iv) 140

(e) How many students finished the race within 1 minute?

(i) 18 (ii) 37 (iii) 31 (iv) 8

(a) (iii) 43 (b) (iii) 60 (c) (ii) Median (d) (iii) 80 (e) (iii) 31

2 marks questions with sample solutions…

1 3 bells ring at an interval of 4, 7 and 14 minutes. All three bells rang at 6 am, when the three bells will ring together next? (2)

Here $4 = 2 \times 2$

$$7 = 7 \times 1$$

$$14 = 2 \times 7$$

$$\text{LCM of } 4, 7, 14 = 2 \times 2 \times 7 = 28$$

$\therefore$ The three bells will ring together again at 6 : 28 am.

2 Find the point on x-axis which is equidistant from the points (2, – 2) and (– 4, 2).

Let P(x, 0) be a point on x-axis then

$$PA = PB \quad \Rightarrow \quad PA^2 = PB^2$$

$$\Rightarrow \quad (x-2)^2 + (0+2)^2 = (x+4)^2 + (0-2)^2$$

$$\Rightarrow \quad x^2 + 4 - 4x + 4 = x^2 + 16 + 8x + 4$$

$$\Rightarrow \quad -4x + 4 = 8x + 16 \quad \Rightarrow \quad x = -1$$

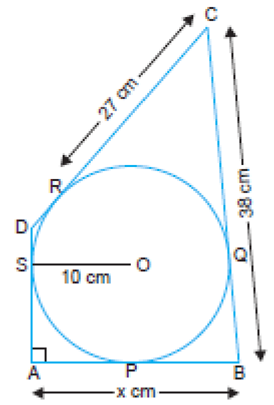
$\Rightarrow$ P (– 1, 0)

3	*P(– 2, 5) and Q(3, 2) are two points. Find the co-ordinates of the point R on PQ such that PR = 2QR.*
	Given PR = 2QR PR : QR = 2 : 1 $R\left(\frac{1(-2)+2(3)}{2+1}, \frac{1(5)+2(2)}{2+1}\right)$ $R\left(\frac{4}{3}, 3\right)$
4	*Find a quadratic polynomial whose zeroes are $5-3\sqrt{2}$ and $5+3\sqrt{2}$.*
	Here Sum of zeroes = $5-3\sqrt{2}+5+3\sqrt{2}=10$ and Product of zeroes = $(5-3\sqrt{2})(5+3\sqrt{2})=7$ Required polynomial = x^2 – (Sum of zeroes)x + product of zeroes $\Rightarrow$ $P(x) = x^2 - 10x + 7$
5	*Draw a line segment AB of length 9 cm. With A and B as centres, draw circles of radius 5 cm and 3 cm respectively. Construct tangents to each circle from the centre of the other circle. (2)*
6	*If tan A = 3/4, find the value of 1/sin A + 1/cos A.*
	$\tan A = \frac{3}{4} = \frac{3k}{4k}$ $\sin A = \frac{3k}{5k} = \frac{3}{5}$, $\cos A = \frac{4k}{5k} = \frac{4}{5}$ $\frac{1}{\sin A} + \frac{1}{\cos A} = \frac{5}{3} + \frac{5}{4} = \frac{(20+15)}{12} = \frac{35}{12}$
7	*If $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0° < \theta < 90°$, find the value of θ.*

$$\sqrt{3}\sin\theta = \cos\theta \quad\Rightarrow\quad \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}} \quad\Rightarrow\quad \theta = 30°$$

8 *In the figure, quadrilateral ABCD is circumscribing a circle with centre O and AD ⊥ AB. If radius of incircle is 10 cm, then the value of x is* (2)



$$\angle A = \angle OPA = \angle OSA = 90°$$

Hence, $\angle SOP = 90°$

Also, $AP = AS$

Hence, OSAP is a square

$$AP = AS = 10 \text{ cm}$$

$$CR = CQ = 27 \text{ cm}$$

$$BQ = BC - CQ = 38 - 27 = 11 \text{ cm}$$

$$BP = BQ = 11 \text{ cm}$$

$$x = AB = AP + BP = 10 + 11 = 21 \text{ cm}$$

PRACTICE… 03 MARKS

1 The diameters of the lower and upper ends of a bucket in the form of a frustum of a cone are 10 cm and 30 cm respectively. If its height is 24 cm, find:

(i) The area of the metal sheet used to make the bucket.

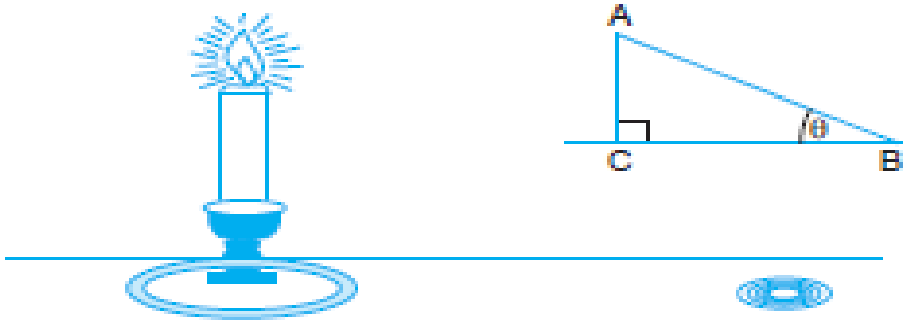
(ii) Why we should avoid the bucket made by ordinary plastic?

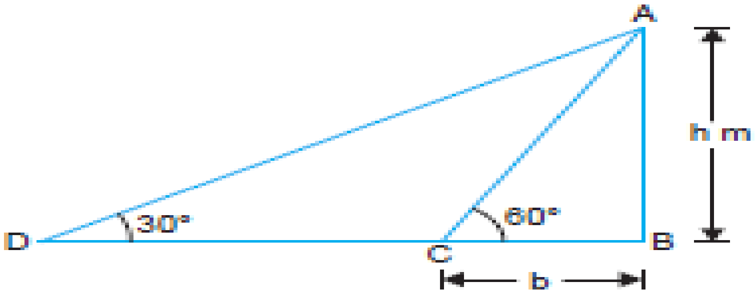
2 *Prove that $2-\sqrt{3}$ is irrational, given that $\sqrt{3}$ is irrational.*

3	*If one root of the quadratic equation $3x^2 + px + 4 = 0$ is 2/3, then find the value of p and the other root of the equation.* (3)
4	*The roots α and β of the quadratic equation $x^2 - 5x + 3(k - 1) = 0$ are such that $\alpha - \beta = 1$. Find the value of k.*
5	*In the figure, ABCD is a square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region.* (3)
6	*The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm, find the length of the corresponding side of the second triangle.* (3)
7	*In an equilateral triangle ABC, D is a point on side BC such that BD = 1/3 BC. Prove that $9AD^2 = 7AB^2$.*
8	*The median of the following data is 16. Find the missing frequencies a and b if the total of the frequencies is 70.* (3)

Class	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40
Frequency	12	a	12	15	b	6	6	4

9 ***If the angles of elevation of the top of the candle from two coins distant 'a' cm and 'b' cm (a > b) from its base and in the same straight line from it are 30° and 60°, then find the height of the candle.*** (3)



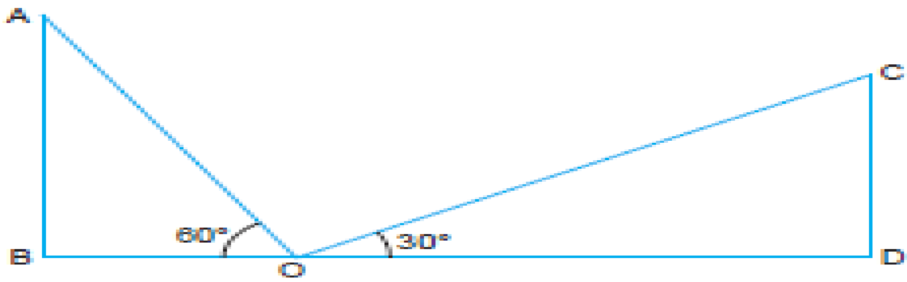


Three/Five marks questions for practice…

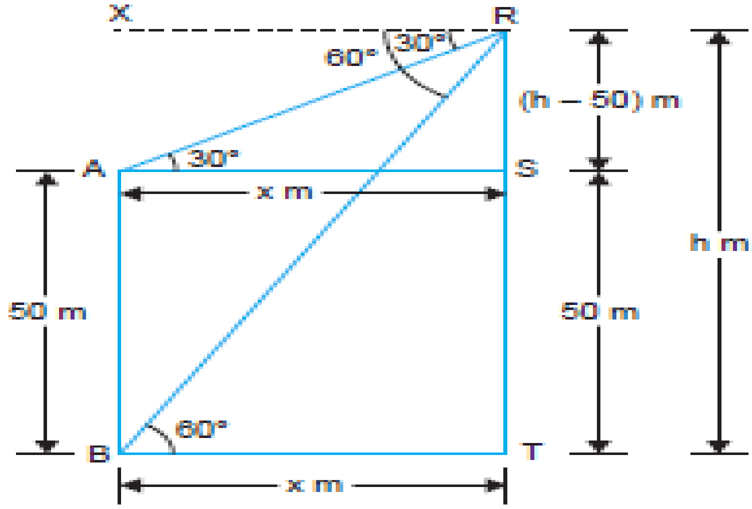
1 *The mode of the following data is 67. Find the missing frequency x.*

Class	40–50	50–60	60–70	70–80	80–90
Frequency	5	x	15	12	7

2 *The two palm trees are of equal heights and are standing opposite each other on either side of the river, which is 80 m wide. From a point O between them on the river the angles of elevation of the top of the trees are 60° and 30°, respectively. Find the height of the trees and the distances of the point O from the trees.* (5)



3 *The angles of depression of the top and bottom of a building 50 meters high as observed from the top of a tower are 30° and 60° respectively. Find the height of the tower, and also the horizontal distance between the building and the river.*

	
4	*Water is flowing through a cylindrical pipe of internal diamter 2 cm, into a cylindrical tank of base radius 40 cm at the rate of 0.7 m/sec. By how much will the water rise in the tank in half an hour?* (5)
5	*A motorboar covers a distance of 16 km upstream and 24 km downstream in 6 hours. In the same time it covers a distance of 12 km upstream and 36 km downstream. Find the speed of the boat in still water and that of the stream.* (5)
6	*A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 1 hr less for the same journey. Find the speed of the train.*
7	*If the sum of the first p terms of an A. P. is q and the sum of the first q terms is p; then show that the sum of the first (p + q) terms is {- (p + q)}.*

8 In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite to the first side is a right angle.

9 Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the isosceles triangle.

10 A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of a 20 m high building, find the elevation of the same bird to be 45°. The boy and the girl are on the opposite sides of the bird. Find the distance of the bird from the girl. (Given $\sqrt{2} = 1.414$)

11 Find the values of frequencies x and y in the following frequency distribution table, if $N = 100$ and median is 32.

Marks:	0-10	10-20	20-30	30-40	40-50	50-60	Total
No. of Students:	10	x	25	30	y	10	100

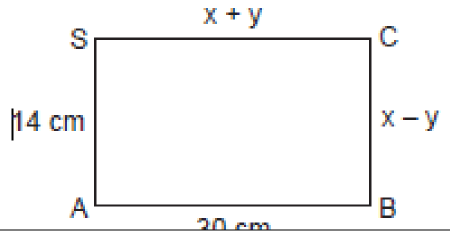
12 **Prove that:**

$$\frac{(1 + \cot\theta + \tan\theta)(\sin\theta - \cos\theta)}{(\sec^3\theta - \mathrm{cosec}^3\theta)} = \sin^2\theta\cos^2\theta$$

13 An open metallic bucket is in the shape of a frustum of a cone. If the diameters of the two circular ends of the bucket are 45 cm and 25 cm and the vertical height of the bucket is 24 cm, find the area of the metallic sheet used to make the bucket. Also find the volume of the water it can hold.

14	Given $\Delta ABC \sim \Delta PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then find $\frac{ar\,\Delta ABC}{ar\,\Delta PQR}$
15	What is the value of $(\cos^2 67^\circ - \sin^2 23^\circ)$?
16	Find the distance of a point $P(x, y)$ from the origin.
17	If $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of k.
18	What is the HCF of smallest prime number and the smallest composite number?
19	In an AP, if the common difference $(d) = -4$ and the seventh term (a_7) is 4, then find the first term.
20	An integer is chosen random between 1 and 100. Find the probability that it is: (*i*) divisible by 8. (*ii*) not divisible by 8.
21	Two different dice are tossed together. Find the probability (*i*) of getting a doublet (*ii*) of getting a sum 10, of the numbers on the two dice.
22	Find the ratio in which $P(4, m)$ divides the line segment joining the points $A(2, 3)$ and $B(6, -3)$. Hence find m.
23	Given that $\sqrt{2}$ is irrational, prove that $(5 + 3\sqrt{2})$ is an irrational number.

24 In Fig., ABCD is a rectangle. Find the values of x and y.



25 **Find the sum of first 8 multiples of 3.**

26 A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed. (3)

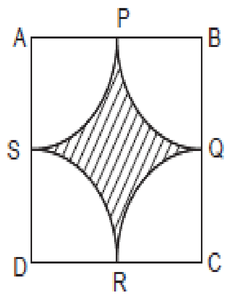
27 Prove that the area of an equilateral triangle described on one side of the square is equal to half the area of the equilateral triangle described on one of its diagonal. (3)

28 Prove that the lengths of tangents drawn from an external point to a circle are equal.

29 A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. Find the total surface area of the article.

30 If $4\tan\theta = 3$, evaluate $\left(\dfrac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right)$.

31 Find the area of the shaded region in Fig., where arcs drawn with centers A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD and DA respectively of a square ABCD of side 12 cm. [Use $\pi = 3.14$] (3)



32	If $A(-2,1)$, $B(a,0)$, $C(4,b)$ and $D(1,2)$ are the vertices of a parallelogram ABCD, find the values of a and b. Hence find the lengths of its sides.
33	Find HCF and LCM of 404 and 96 and verify that HCF $\times$ LCM = Product of the two given numbers.
34	Find all zeroes of the polynomial $(2x^4-9x^3+5x^2+3x-1)$ if two of its zeroes are $(2+\sqrt{3})$ and $(2-\sqrt{3})$.
35	Draw a triangle ABC wih BC = 6 cm, AB = 5 cm and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the ΔABC.
36	The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7 : 15. Find the numbers.
37	In an equilateral ΔABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9(AD)^2 = 7(AB)^2$.
38	A motor boat whose speed is 18 km/hr in still water takes 1 hr more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.
39	As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between two ships.
40	The diameters of the lower and upper ends of a bucket in the form of a frustum of a cone are 10 cm and 30 cm respectively. If its height is 24 cm, find: (*i*) The area of the metal sheet used to make the bucket. (*ii*) Why we should avoid the bucket made by ordinary plastic?

41	The mean of the following distribution is 18. Find the frequency f of the class 19–21. Class: 11-13, 13-15, 15-17, 17-19, 19-21, 21-23, 23-25 Frequency: 3, 6, 9, 13, f, 5, 4
42	Prove that: $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$
43	If the area of two similar triangles are equal, prove that they are congruent.
44	A heap of rice is in the form of a cone of base diameter 24 m and height 2.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap? (3)
45	If $\tan 2A = \cot (A - 18^\circ)$, where $2A$ is an acute angle, find the value of A.
46	If $A(-5, 7)$, $B(-4, -5)$, $C(-1, -6)$ and $D(4, 5)$ are the vertices of a quadrilateral ABCD, find the area of the quadrilateral ABCD.
47	Prove that, in a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.
48	A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete total journey, what is the original average speed?

Class	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

49	**The table below shows the salaries of 280 persons:**

Salary (In thousand ₹)	*(No. of persons)*
5–10	49
10–15	133
15–20	63
20–25	15
25–30	6
30–35	7
35–40	4
40–45	2
45–50	1

Calculate the median salary of the data.

50 A heap of rice is in the form of a cone of base diameter 24 m and height 2.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap? (3)

51 *The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the plane is flying at a constant height of 3600 $\sqrt{3}$ metres, find the speed of the aeroplane.*

52 *Solve for x :*

$$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x};\ a \neq b \neq 0,\ x \neq 0,\ x \neq -(a+b)$$

53 *A girl empties a cylindrical bucket full of sand, of base radius 18 cm and height 32 cm on the floor to form a conical heap of sand. If the height of this conical heap is 24 cm, then find its slant height correct to one place of decimal.*

54 *In Figure, E is a point on CB produced of an isosceles ΔABC, with side $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\Delta ABD \sim \Delta ECF$.*

55	*If* $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, *show that* $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.
56	*For what value of p, are the points (2, 1), (p, – 1) and (–1, 3) collinear ?*
57	*Show that any positive odd integer is of the form 6m + 1 or 6m + 3 or 6m + 5, where m is some integer.*
58	*If the* 17^{th} *term of an A.P. exceeds its* 10^{th} *term by 7, find the common difference.*

BEST OF LUCK

Class- X

Mathematics-Basic (241)

Sample Question Paper 2020-21

Max. Marks: 80 **Duration:3 hours**

General Instructions:

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

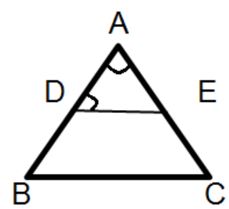
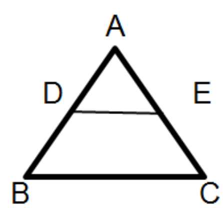
Part – A:

1. It consists of two sections- I and II
2. Section I has 16 questions. Internal choice is provided in 5 questions.
3. Section II has four case study-based questions. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B:

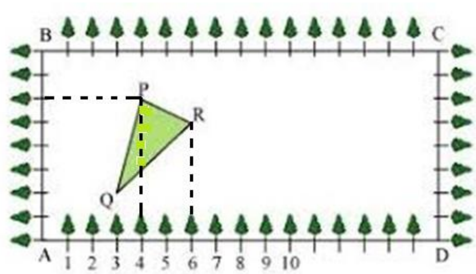
1. Question No 21 to 26 are Very short answer Type questions of 2 mark each,
2. Question No 27 to 33 are Short Answer Type questions of 3 marks each
3. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Question No.	**Part-A**	**Marks**
	Section-I	
1.	Express 156 as the product of primes.	1
2.	Write a quadratic polynomial, sum of whose zeroes is 2 and product is -8.	1
3.	Given that HCF (96,404) is 4, find the LCM (96,404). **OR** State the fundamental Theorem of Arithmetic.	1

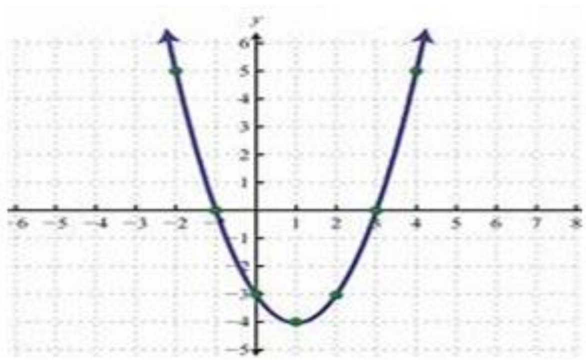
4	On comparing the ratios of the coefficients, find out whether the pair of equations $x - 2y = 0$ and $3x + 4y - 20 = 0$ is consistent or inconsistent.	1
5	If a and b are co-prime numbers, then find the HCF (a, b).	1
6	Find the area of a sector of a circle with radius 6cm if angle of the sector is 60°. (Take π = 22/7) **OR** A horse tied to a pole with 28m long rope. Find the perimeter of the field where the horse can graze. (Take π = 22/7)	1
7	In the given fig. DE || BC, ∟ADE =70° and ∟BAC=50°, then angle ∟BCA = ______  **OR** In the given figure, AD = 2cm, BD = 3 cm, AE = 3.5 cm and AC = 7 cm. Is DE parallel to BC ?  	1

8	The cost of fencing a circular field at the rate of Rs.24 per metre is Rs. 5280. Find the radius of the field.	1
9	A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground where it makes an angle 30° . The distance between the foot of the tree to the point where the top touches the ground is 8m. Find the height of the tree from where it is broken.	1
10	If the perimeter and the area of a circle are numerically equal, then find the radius of the circle	1
11	Write the empirical relationship among mean, median and mode.	1
12	To divide a line segment BC internally in the ratio 3 : 5, we draw a ray BX such that ∠CBX is an acute angle. What will be the minimum number of points to be located at equal distances, on ray BX?	1
13	For what values of p does the pair of equations 4x + p y +8 =0 and 2x +2y +2 =0 has unique solution? **OR** What type of straight lines will be represented by the system of equations 2x + 3y =5 and 4x + 6y = 7 ?	1
14	A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is red? **OR** A die is thrown once. What is the probability of getting a prime number?	1
15	A tower stands vertically on the ground. From a point on the ground, which is 15m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°.Find the height of the tower.	1
16	Probability of an event E + Probability of the event $\overline{\mathrm{E}}$ (not E) is,_______	1

	Section-II **Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark**	
17	 Mathematics teacher of a school took her 10th standard students to show Red fort. It was a part of their Educational trip. The teacher had interest in history as well. She narrated the facts of Red fort to students. Then the teacher said in this monument one can find combination of solid figures. There are 2 pillars which are cylindrical in shape. Also 2 domes at the corners which are hemispherical.7 smaller domes at the centre. Flag hoisting ceremony on Independence Day takes place near these domes.	
i)	How much cloth material will be required to cover 2 big domes each of radius 2.5 metres? (Take π = 22/7) a) $75m^2$ b) $78.57m^2$ c) $87.47m^2$ d) $25.8m^2$ b)	1
ii)	Write the formula to find the volume of a cylindrical pillar. a) $\Pi r^2 h$ b) Πrl c) $\Pi r(l + r)$ d) $2\Pi r$	1
iii)	Find the lateral surface area of two pillars if height of the pillar is 7m and radius of the base is 1.4m. a) $112.3cm^2$ b) $123.2m^2$ c) $90m^2$ d) $345.2cm^2$	1
iv)	How much is the volume of a hemisphere if the radius of the base is 3.5m? a) $85.9\ m^3$ b) $80\ m^3$ c) $98\ m^3$ d) $89.83\ m^3$	1

v)	What is the ratio of sum of volumes of two hemispheres of radius 1cm each to the volume of a sphere of radius 2 cm? a) 1:1 b) 1:8 c) 8 :1 d) 1:16	1
18	Class X students of a secondary school in Krishnagar have been allotted a rectangular plot of a land for gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1m from each other. There is a triangular grassy lawn in the plot as shown in the fig. The students are to sow seeds of flowering plants on the remaining area of the plot.  Considering A as origin, answer question (i) to (v)	
i)	Considering A as the origin, what are the coordinates of A? a) (0,1) b) (1,0) c) (0,0) d)(-1,-1)	1
ii)	What are the coordinates of P? a) (4,6) b)(6,4) c) (4,5) d) (5,4)	1
iii)	What are the coordinates of R? a) (6,5) b) (5,6) c) (6,0) d) (7,4)	1
iv)	What are the coordinates of D? a) (16,0) b) (0,0) c) (0,16) d) (16,1)	1
v)	What are the coordinate of P if D is taken as the origin? a) (12,2) b) (-12,6) c) (12,3) d) (6,10)	1

19	 Rahul is studying in X Standard. He is making a kite to fly it on a Sunday. Few questions came to his mind while making the kite. Give answers to his questions by looking at the figure.	
i)	Rahul tied the sticks at what angles to each other? a) 30° b) 60° c) 90° d) 60°	1
ii)	Which is the correct similarity criteria applicable for smaller triangles at the upper part of this kite? a) RHS b) SAS c) SSA d) AAS	1
iii)	Sides of two similar triangles are in the ratio 4:9. Corresponding medians of these triangles are in the ratio, a) 2:3 b) 4:9 c) 81:16 d) 16:81	1
iv)	In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle. This theorem is called as, a) Pythagoras theorem b) Thales theorem c) Converse of Thales theorem d) Converse of Pythagoras theorem	1
v)	What is the area of the kite, formed by two perpendicular sticks of length 6 cm and 8 cm? a) 48 cm^2 b) 14 cm^2 c) 24 cm^2 d) 96 cm^2	1

20	Due to heavy storm an electric wire got bent as shown in the figure. It followed a mathematical shape. Answer the following questions below. 	
i)	Name the shape in which the wire is bent a) Spiral b) ellipse c) linear d) Parabola	1
ii)	How many zeroes are there for the polynomial (shape of the wire) a) 2 b) 3 d) 1 d) 0	1
iii)	The zeroes of the polynomial are a) -1, 5 b) -1, 3 c) 3, 5 d) -4, 2	1
iv)	What will be the expression of the polynomial? a) x^2+2x-3 b) x^2-2x+3 c) x^2-2x-3 d) x^2+2x+3	1
v)	What is the value of the polynomial if x = -1? a) 6 b) -18 c)) 18 d) 0	1
	Part –B **All questions are compulsory. In case of internal choices, attempt anyone.**	
21	Find the coordinates of the point which divides the line segment joining the points (4, -3) and (8,5) in the ratio 3:1 internally.	2

<table>
<tr><td></td><td> OR Find a relation between x and y such that the point (x,y) is equidistant from the points (7,1) and (3,5) </td><td></td></tr>
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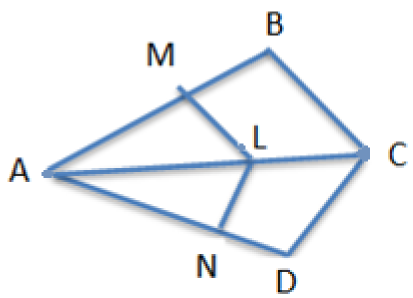

In the fig. if LM II CB and LN II CD, prove that $\frac{AM}{MB} = \frac{AN}{ND}$

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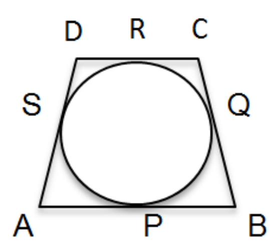
A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.



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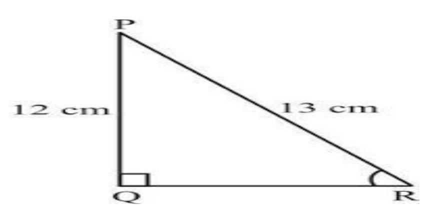
<tr><td>24</td><td>Draw a line segment of length 7.8 cm and divide it in the ratio 5:8. Measure the two parts.</td><td>2</td></tr>
<tr><td>25</td><td>

Given 15 cot A =8, find sin A and sec A.

OR

Find tan P – cot R



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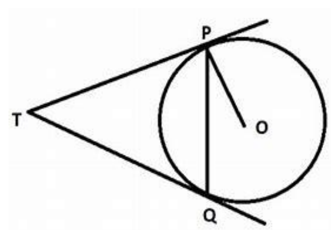
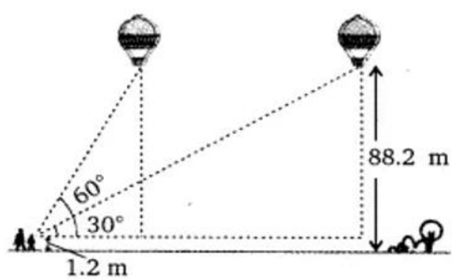
26	How many terms of the A. P : 9,17,25, ...must be taken to give a sum 636?	2
	Part –B **All questions are compulsory. In case of internal choices, attempt anyone.**	
27	Prove that $\sqrt{3}$ is an irrational number.	3
28	Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∟PTQ = 2∟OPQ.  	3
29	Meena went to a bank to withdraw Rs.2,000. She asked the cashier to give her Rs.50 and Rs.100 notes only. Meena got 25 notes in all. Find how many notes of Rs.50 and Rs.100 she received.	3
30	A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number. (iii) a number divisible by 5. **OR** One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting (i) A king of red colour. (ii) A spade (iii) The queen of diamonds	3
31	Metallic spheres of radii 6cm, 8cm and 10cm respectively are melted to form a solid sphere. Find the radius of the resulting sphere.	3

32	Prove that $\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \frac{1}{\sec A - \tan A}$	3
33	A motor boat whose speed in still water is 18 km/h, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream. **OR** Find two consecutive odd positive integers, sum of whose squares is 290.	3
	Part –B **All questions are compulsory. In case of internal choices, attempt anyone.**	
34	The angles of depression of the top and bottom of a 8m tall building from the top of a multi storied building are 30° and 45°, respectively. Find the height of the multi storied building and the distance between the two buildings. **OR** A 1.2m tall girl spots a balloon moving with the wind in a horizontal line at a height 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°.After sometime, the angle of elevation reduces 30°.Find the distance travelled by the balloon during the interval.  	5
35	The p^{th}, q^{th} and r^{th} terms of an A.P. are a, b and c respectively. Show that a(q – r) + b(r-p) + c(p – q) = 0	5

36	A survey regarding the heights in (cm) of 51 girls of class X of a school was conducted and the following data was obtained. Find the median height and the mean using the formulae.	5

Height (in cm)	**Number of Girls**
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

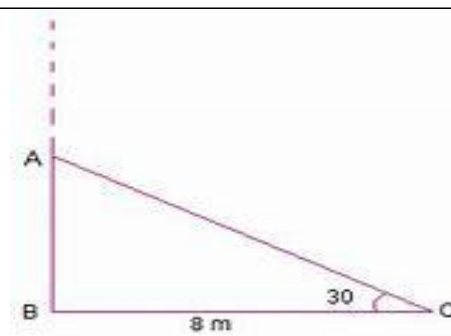
Class- X

Mathematics-Basic (241)

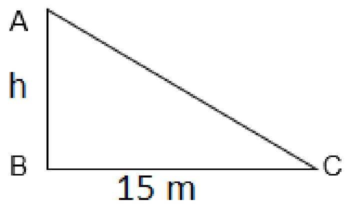
Marking Scheme SQP-2020-21

Max. Marks: 80 **Duration:3hrs**

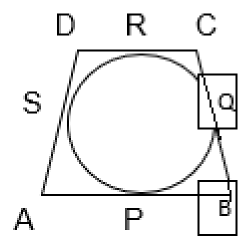
1	$156 = 2^2 \times 3 \times 13$	1
2	Quadratic polynomial is given by $x^2 - (a+b)x + ab$ $x^2 - 2x - 8$	1
3	HCF X LCM =product of two numbers LCM (96,404) $= \frac{96 \, X \, 404}{HCF(96,404)} = \frac{96 \, X \, 404}{4}$ LCM = 9696 **OR** Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the factors occur.	½ ½ 1
4	$x - 2y = 0$ $3x + 4y - 20 = 0$ $\frac{1}{3} \neq \frac{-2}{4}$ As, $\frac{a1}{a2} \neq \frac{b1}{b2}$ is one condition for consistency. Therefore, the pair of equations is consistent.	 ½ ½
5	1	1
6	$\theta = 60°$ Area of sector $= \frac{\theta}{360°} \Pi r^2$ $A = \frac{60°}{360°} \, X \, \frac{22}{7} \, X \, (6)^2 \, cm^2$ $A = \frac{1}{6} \, X \, \frac{22}{7} \, X 36 \, cm^2$ $= 18.86 cm^2$	 ½ ½

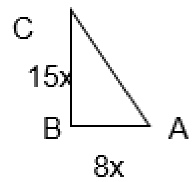
	OR Another method- Horse can graze in the field which is a circle of radius 28 cm. So, required perimeter = $2\Pi r = 2.\Pi(28)$ cm $= 2 \times \frac{22}{7} \times (28)$cm = 176 cm	 ½ ½
7	By converse of Thale's theorem DE II BC $\angle ADE = \angle ABC = 70°$ Given $\angle BAC = 50°$ $\angle ABC + \angle BAC + \angle BCA = 180°$ (Angle sum prop of triangles) $70^0 + 50^0 + \angle BCA = 180°$ $\angle BCA = 180° - 120° = 60°$ **OR** EC = AC – AE = (7- 3.5) cm = 3.5 cm $\frac{AD}{BD} = \frac{2}{3}$ and $\frac{AE}{EC} = \frac{3.5}{3.5} = \frac{1}{1}$ So, $\frac{AD}{BD} \neq \frac{AE}{EC}$ Hence, By converse of Thale's Theorem, DE is not Parallel to BC.	 ½ ½ ½ ½
8	Length of the fence = $\frac{Total\ cost}{Rate}$ $= \frac{Rs.5280}{Rs\ 24/metre}$ = 220 m So, length of fence = Circumference of the field ∴ 220m= $2\ \Pi\ r = 2 \times \frac{22}{7} \times r$ So, r = $\frac{220\ x\ 7}{2\ \ x\ 22}$ m =35 m	 ½ ½
9	 Sol: tan 30 ° $= \frac{AB}{BC}$ $1/\sqrt{3} = \frac{AB}{8}$ AB = 8 / √3 metres Height from where it is broken is 8/√3 metres	 ½ ½

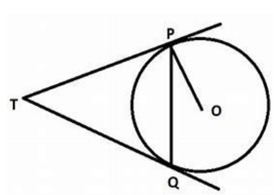
10	Perimeter = Area $2\Pi r = \Pi r^2$ r = 2 units	1
11	3 median = mode + 2 mean	1
12	8	1
13	$\frac{a1}{a2} \neq \frac{b1}{b2}$ is the condition for the given pair of equations to have unique solution. $\frac{4}{2} \neq \frac{p}{2}$ $p \neq 4$ Therefore, for all real values of p except 4, the given pair of equations will have a unique solution. **OR** Here, $\frac{a1}{a2} = \frac{2}{4} = \frac{1}{2}$ $\frac{b1}{b2} = \frac{3}{6} = \frac{1}{2}$ and $\frac{c1}{c2} = \frac{5}{7}$ $\frac{1}{2} = \frac{1}{2} \neq \frac{5}{7}$ $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$ is the condition for which the given system of equations will represent parallel lines. So, the given system of linear equations will represent a pair of parallel lines.	½ ½ ½ ½
14	No. of red balls = 3, No.black balls =5 Total number of balls = 5 + 3 =8 Probability of red balls =$\frac{3}{8}$ **OR** Total no of possible outcomes = 6 There are 3 Prime numbers, 2,3,5. So, Probability of getting a prime number is $\frac{3}{6} = \frac{1}{2}$	½ ½ ½ ½

15	 tan $60° = \frac{h}{15}$ $\sqrt{3} = \frac{h}{15}$ $h = 15\sqrt{3}$ m	½ ½
16	1	1
17 i)	Ans : b) Cloth material required = 2X S A of hemispherical dome $= 2 \times 2\Pi r^2$ $= 2 \times 2 \times \frac{22}{7} \times (2.5)^2 \, m^2$ $= 78.57 \; m^2$	1
ii)	a) Volume of a cylindrical pillar = $\Pi r^2 h$	1
iii)	b) Lateral surface area = 2x $2\Pi rh$ $= 4 \times \frac{22}{7} \times 1.4 \times 7 \, m^2$ $= 123.2 \, m^2$	1
iv)	d) Volume of hemisphere $= \frac{2}{3} \Pi r^3$ $= \frac{2}{3} \frac{22}{7} (3.5)^3 \, m^3$ $= 89.83 \; m^3$	1
v)	b) Sum of the volumes of two hemispheres of radius 1cm each= $2 \times \frac{2}{3} \Pi 1^3$ Volume of sphere of radius 2cm = $\frac{4}{3} \Pi \, 2^3$ So, required ratio is $\frac{2 \times \frac{2}{3} \Pi \, 1^3}{\frac{4}{3} \Pi \; 2^3} =$ 1:8	½ ½

18 i)	c) (0,0)	1
ii)	a) (4,6)	1
iii)	a) (6,5)	1
iv)	a) (16,0)	1
v)	b) (-12,6)	1
19 i)	c) 90°	1
ii)	b) SAS	1
iii)	b) 4 : 9	1
iv)	d) Converse of Pythagoras theorem	1
v)	a) 48 cm^2	1
20 i)	d) parabola	1
ii)	a) 2	1
iii)	b) -1, 3	1
iv)	c) $x^2 - 2x - 3$	1
v)	d) 0	1
21	Let P(x,y) be the required point. Using section formula $\{\frac{m\,1x2+m2x1}{m1+m2}, \frac{m1y2+m2y1}{m1+m2}\}$ = (x, y) x = $\frac{3(8)+1(4)}{3+1}$, y = $\frac{3(5)+1(-3)}{3+1}$ x = 7 y= 3 (7,3) is the required point	1 1

	OR Let P(x, y) be equidistant from the points A(7,1) and B(3,5) Given AP =BP. So, $AP^2 = BP^2$ $(x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2$ $x^2 -14x+49 +y^2-2y +1 = x^2 -6x +9+y^2 -10y+25$ $x - y =2$	1 1
22	By BPT, $\frac{AM}{MB} = \frac{AL}{LC}$ ....(1) Also, $\frac{AN}{ND} = \frac{AL}{LC}$ ....(2) By Equating (1) and (2) $\frac{AM}{MB} = \frac{AN}{ND}$	½ ½ 1
23	To prove: AB + CD = AD + BC.  Proof: AS = AP (Length of tangents from an external point to a circle are equal) BQ = BP CQ = CR DS = DR AS + BQ + CQ + DS = AP + BP + CR + DR (AS+ DS) + (BQ + CQ) = (AP + BP) + (CR + DR) AD + BC = AB +CD	1 1
24	For the correct construction	2

25	15 cot A =8, find sin A and sec A. Cot A =8/15  $\frac{Adj}{Oppo}$ =8/15 By Pythagoras Theorem $AC^2 = AB^2 + BC^2$ AC =$\sqrt{(8x)^2 + (15x)^2}$ AC= 17x Sin A = 15/17 Cos A =8/17 **OR** By Pythagoras Theorem QR = $\sqrt{(13)^2 - (12)^2}$ cm QR = 5cm Tan P =5/12 Cot R =5/12 Tan P -Cot R =5/12 -5/12 = 0	1 ½ ½ 1 1
26	9,17,25, ... $S_n = 636$ a = 9 d = $a_2 - a_1$ = 17 – 9 = 8 $S_n = \frac{n}{2}$ [2a + (n-1) d] Sn = $\frac{n}{2}$ [2a + (n-1) d]	½ ½

	$636 = \frac{n}{2}$ [2x 9 + (n-1) 8] 1272 = n [18 + 8n -8] 1272 = n [10 +8n] $8n^2$ +10n -1272 =0 $4n^2$ + 5n -636 =0 $n = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $n = \frac{-5\pm\sqrt{5^2-4x\ 4x(-636)}}{2x4}$ $n =-\frac{-5\pm101}{8}$ $n=\frac{96}{8}$ n=12 $n =\frac{-106}{8}$ $n = -\frac{-53}{4}$ n=12 (since n cannot be negative)	 ½ ½
27	Let √3 be a rational number. Then √3 = p/q HCF (p,q) =1 Squaring both sides $(\sqrt{3})^2 = (p/q)^2$ $3 = p^2/ q^2$ $3q^2 = p^2$ 3 divides p^2 » 3 divides p 3 is a factor of p Take p = 3C $3q^2 = (3c)^2$ $3q^2 = 9C^2$ 3 divides q^2 » 3 divides q 3 is a factor of q Therefore 3 is a common factor of p and q It is a contradiction to our assumption that p/q is rational. Hence √3 is an irrational number.	 1 ½ ½ 1
28		

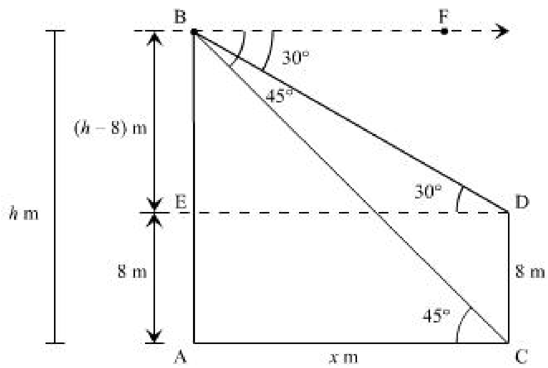
	Required to prove -: ∟PTQ = 2∟OPQ Sol :- Let ∟PTQ = θ Now by the theorem TP = TQ. So, TPQ is an isosceles triangle ∟TPQ = ∟TQP = ½ (180° -θ) = 90° - ½ θ ∟OPT = 90° ∟OPQ =∟OPT -∟TPQ =90° -(90° - ½ θ) = ½ θ = ½ ∟PTQ ∟PTQ = 2∟OPQ	1 1 ½ ½
29	Let Meena has received x no. of 50 re notes and y no. of 100 re notes.So, 50 x + 100 y =2000 x + y =25 multiply by 50 50x + 100y =2000 50 x + 50 y = 1250 - - - 50y =750 Y= 15 Putting value of y=15 in equation (2) x+ 15 =25 x = 10 Meena has received 10 pieces 50 re notes and 15 pieces of 100 re notes	1 1 1
30	(i) 10,11,12…90 are two digit numbers. There are 81 numbers.So,Probability of getting a two-digit number = 81/90 =9/10 (ii) 1, 4, 9,16,25,36,49,64,81 are perfect squares. So, Probability of getting a perfect square number. = 9/90 =1/10 (iii) 5, 10,15….90 are divisible by 5. There are 18 outcomes.. So,Probability of getting a number divisible by 5. = 18/90 =1/5	1 1 1

	OR (i) Probability of getting A king of red colour. P (King of red colour) = 2/52 =1/26 (ii) Probability of getting A spade P (a spade) = 13/52 = 1/4 (iii) Probability of getting The queen of diamonds P (a the queen of diamonds) = 1/52	 1 1 1
31	$r_1 = 6cm$ $r_2 = 8cm$ $r_3 = 10cm$ Volume of sphere = $^4/_3 \Pi r^3$ Volume of the resulting sphere = Sum of the volumes of the smaller spheres. $^4/_3 \Pi r^3 = {^4/_3} \Pi r_1^3 + {^4/_3} \Pi r_2^3 + {^4/_3} \Pi r_3^3$ $^4/_3 \Pi r^3 = {^4/_3} \Pi (r_1^3 + r_2^3 + r_3^3)$ $r^3 = 6^3 + 8^3 + 10^3$ $r^3 = 1728$ $r = \sqrt[3]{1728}$ $r = 12$ cm Therefore, the radius of the resulting sphere is 12cm.	 1 1 1
32	(sin A-cos A+1)/ (sin A+cosA-1) = 1/(sec A-tan A) L.H.S. divide numerator and denominator by cos A = (tan A-1+secA)/ (tan A+1-sec A) = (tan A-1+secA)/(1-sec A + tan A) We know that $1+\tan^2 A=\sec^2 A$ Or $1=\sec^2 A-\tan^2 A$ = (sec A + tan A)(sec A – tan A) =(sec A + tan A-1)/[(sec A + tan A)(sec A-tan A)-(sec A-tan A)] =(sec A + tan A-1)/(sec A-tan A)(sec A + tan A-1)	 1 1 1

	= 1/(sec A-tan A) , proved.	
33	Given:- Speed of boat =18*km*/*hr* Distance =24*km* Let *x* be the speed of stream. Let *t*1 and *t*2 be the time for upstream and downstream. As we know that, speed= distance / time ⇒time= distance / speed For upstream, Speed =(18−*x*) *km*/*hr* Distance =24*km* Time =*t*1 Therefore, $t_1 = \frac{24}{18-x}$ For downstream, Speed =(18+*x*)*km*/*hr* Distance =24*km* Time =*t*2 Therefore, $t_2 = \frac{24}{18+x}$ Now according to the question- *t*1=*t*2+1 $\frac{24}{18-x} = \frac{24}{18+x} + 1$ $\Rightarrow \frac{24(18+x) - 24\,(18-x\,)}{(18-x)(18+x)} = 1$ $\Rightarrow 48x=(18-x)(18+x)$ $\Rightarrow 48x=324+18x-18x-x^2$ $\Rightarrow x^2+48x-324=0$ $\Rightarrow x^2+54x-6x-324=0$ $\Rightarrow x(x+54)-6(x+54)=0$ $\Rightarrow (x+54)(x-6)=0$	½ ½ ½

	$\Rightarrow x = -54$ or $x = 6$	½
	Since speed cannot be negative.	
	$\Rightarrow x = -54$ will be rejected	
	$\therefore x = 6$	
	Thus, the speed of stream is 6*km*/*hr.*	1
	OR	
	Let one of the odd positive integer be x	
	then the other odd positive integer is x+2	1
	their sum of squares = $x^2 + (x+2)^2$	
	$= x^2 + x^2 + 4x + 4$	
	$= 2x^2 + 4x + 4$	
	Given that their sum of squares = 290	
	$\Rightarrow 2x^2 + 4x + 4 = 290$	
	$\Rightarrow 2x^2 + 4x = 290 - 4 = 286$	
	$\Rightarrow 2x^2 + 4x - 286 = 0$	1
	$\Rightarrow 2(x^2 + 2x - 143) = 0$	
	$\Rightarrow x^2 + 2x - 143 = 0$	
	$\Rightarrow x^2 + 13x - 11x - 143 = 0$	
	$\Rightarrow x(x+13) - 11(x+13) = 0$	
	$\Rightarrow (x - 11)(x+13) = 0$	
	$\Rightarrow (x-11) = 0$, $(x+13) = 0$	
	Therefore , x = 11 or -13	
	According to question, x is a positive odd integer.	
	Hence, We take positive value of x	1
	So , x = 11 and (x+2) = 11 + 2 = 13	
	Therefore , the odd positive integers are 11 and 13 .	

34



1

Let AB and CD be the multi-storeyed building and the building respectively.

Let the height of the multi-storeyed building= *h* m and

the distance between the two buildings = *x* m.

AE = CD = 8 m [Given]

BE = AB – AE = (*h* – 8) m

and

AC = DE = *x* m [Given]

Also,

∠FBD = ∠BDE = 30° (Alternate angles)

∠FBC = ∠BCA = 45° (Alternate angles) ½

Now,

In Δ ACB,

$$\Rightarrow \tan 45^0 = \frac{AB}{AC}\left[\because \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}\right]$$

$$\Rightarrow 1 = \frac{h}{x}$$

$$\Rightarrow x = h \ldots\ldots(i)$$ 1

In Δ BDE,

$$\Rightarrow \tan 30^0 = \frac{BE}{ED}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h-8}{x}$$

$$\Rightarrow x = \sqrt{3}(h-8)\ldots\ldots\ldots\ldots(ii)$$

1

From (i) and (ii), we get,

h =√3h -8√3

√3h – h =8√3

h (√3 -1) =8√3

h =$\frac{8\sqrt{3}}{\sqrt{3}-1}$

h=$\frac{8\sqrt{3}}{\sqrt{3}-1}$ x$\frac{\sqrt{3}+1}{\sqrt{3}+1}$ 1

h-= 4√3 (√3 +1)

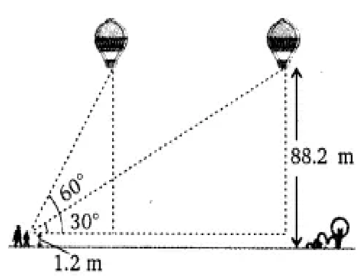
h = 12 +4√3 m

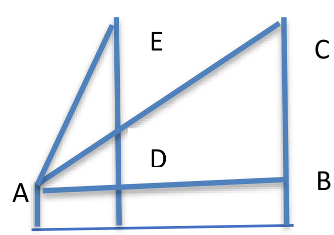
Distance between the two building

$$x = \left(12 + 4\sqrt{3}\right)m \quad [From(i)]$$

½

OR





From the figure, the angle of elevation for the first position of the balloon ∟EAD = 60° and for second position ∟BAC = 30°.The vertical distance 1

ED = CB = 88.2-1.2 =87m.

	Let AD = x m and AB = y m. Then in right Δ ADE, tan60° = $\frac{DE}{AD}$ $\sqrt{3} = \frac{87}{X}$ $X = \frac{87}{\sqrt{3}}$ ……….(i) In right ΔABC, tan 30° = $\frac{BC}{AB}$ $\frac{1}{\sqrt{3}} = \frac{87}{y}$ Y = 87√3 ……….(ii) Subtracting(i) and (ii) y-x =87√3 -- $\frac{87}{\sqrt{3}}$ y-x =$\frac{87 .2.\sqrt{3}}{\sqrt{3}.\sqrt{3}}$ y-x = 58√3 m Hence, the distance travelled by the balloon is equal to BD y-x =58√3 m.	1 1 1 1
35	Let A be the first term and D the common difference of A.P. $Tp=a=A+(p-1)D=(A-D)+pD$ (1) $Tq=b=A+(q-1)D=(A-D)+qD$ ..(2) $Tr=c=A+(r-1)D=(A-D)+rD$ ..(3) Here we have got two unknowns A and D which are to be eliminated. We multiply (1),(2) and (3) by $q-r$,$r-p$ and $p-q$ respectively and add: a (q-r) = (A – D)(q-r) + D p(q-r) b(r-p) = (A-D) (r-p) + Dq (r-p) c(p-q) = (A-D) (p-q) + Dr (p-q) $a(q-r)+b(r-p)+c(p-q)$ $=(A-D)[q-r+r-p+p-q]+D[p(q-r)+q(r-p)+r(p-q)]$ = (A – D) (0) + D [pq-pr + qr – pq + rp – rq) =0	½ ½ ½ ½ ½ ½ 1 1

36	**Height (in cm)** f C.F. below 140 4 4 140-145 7 11 145-150 18 29 150-155 11 40 155-160 6 46 160-165 5 51	1

Height (in cm)	f	C.F.
below 140	4	4
140-145	7	11
145-150	18	29
150-155	11	40
155-160	6	46
160-165	5	51

1

N=51⇒

$N/2$=51/2=25.5

As 29 is just greater than 25.5, therefore median class is 145-150.

$Median = l + \frac{(\frac{N}{2} - C)}{f} X\, h$

Here, l= lower limit of median class =145

C=$C.F.$ of the class preceding the median class =11

½

h= higher limit - lower limit =150−145=5

f= frequency of median class =18

∴*median*=

$= 145 + \frac{(25.5 - 11)}{18} X\, 5$

½

=149.03

Mean by direct method

1

Height (in cm)	f	x_i	fx_i
below 140	4	137.5	550
140-145	7	142.5	997.5
145-150	18	147.5	2655
150-155	11	152.5	1677.5
155-160	6	157.5	945
160-165	5	162.5	812.5

1

$$\text{Mean} = \frac{\sum fx}{N}$$

=7637.5/51

= 149.75

1

Class- X Session- 2020-21

Subject- Mathematics -Standard

Sample Question Paper

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

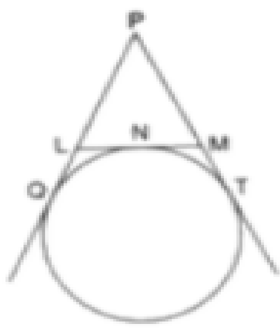
Part – A:

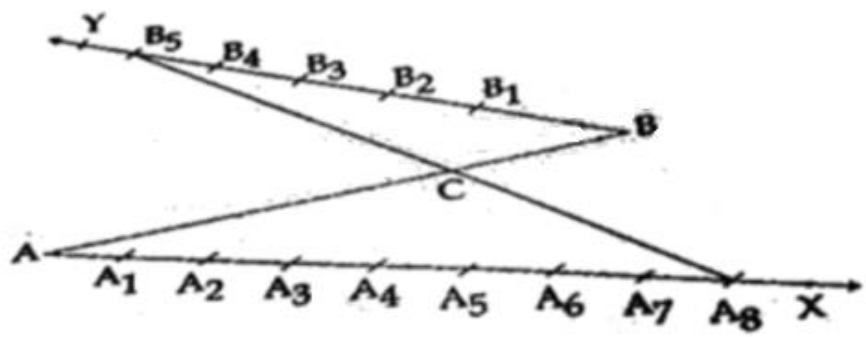
1. It consists three sections- I and II.
2. Section I has 16 questions of 1 mark each. Internal choice is provided in 5 questions.
3. Section II has 4 questions on case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

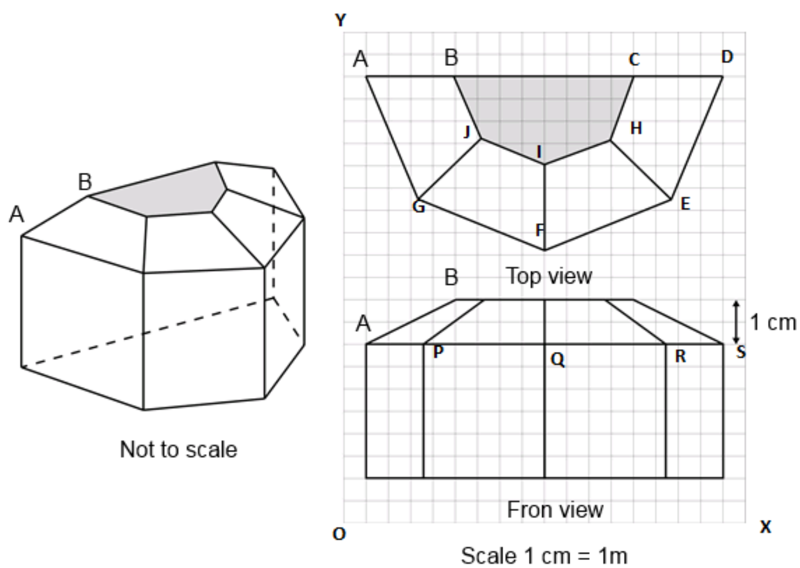
Part – B:

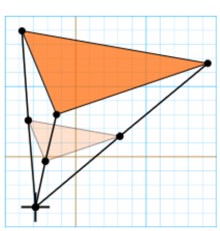
1. Question No 21 to 26 are Very short answer Type questions of 2 mark each,
2. Question No 27 to 33 are Short Answer Type questions of 3 marks each
3. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

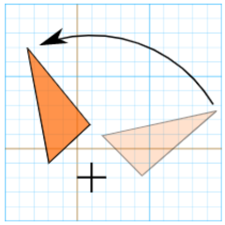
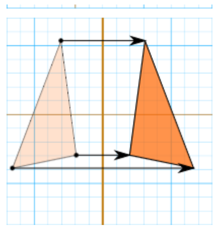
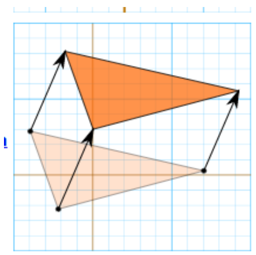
Question No.	Part-A	Marks allocated
	Section-I **Section I has 16 questions of 1 mark each. Internal choice is provided in 5 questions.**	
1	If xy=180 and HCF(x,y)=3, then find the LCM(x,y). **OR** The decimal representation of $\frac{14587}{2^1 \times 5^4}$ will terminate after how many decimal places?	1
2	If the sum of the zeroes of the quadratic polynomial $3x^2$-kx+6 is 3, then find the value of k.	1

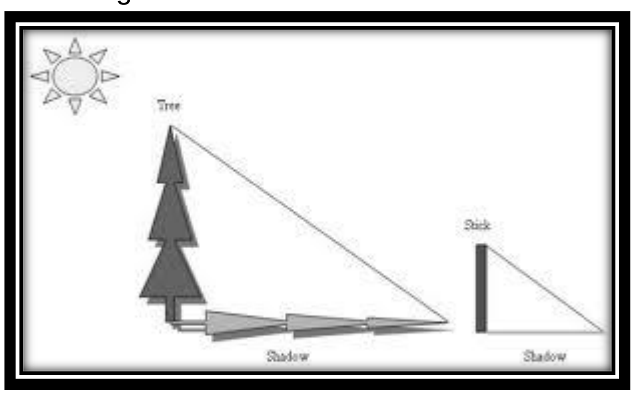
3.	For what value of k, the pair of linear equations 3x+y=3 and 6x+ky=8 does not have a solution.	1
4.	If 3 chairs and 1 table costs Rs. 1500 and 6 chairs and 1 table costs Rs.2400. Form linear equations to represent this situation.	1
5.	Which term of the A.P. 27, 24, 21,…..is zero? **OR** In an Arithmetic Progression, if d= - 4, n=7, a_n=4, then find a.	1
6.	For what values of k, the equation $9x^2+6kx+4=0$ has equal roots?	
7.	Find the roots of the equation $x^2+7x+10=0$ **OR** For what value(s) of 'a' quadratic equation $30\ ax^2 - 6x + 1 = 0$ has no real roots?	1
8.	If PQ=28cm, then find the perimeter of ΔPLM 	1
9.	If two tangents are inclined at 60˚ are drawn to a circle of radius 3cm then find length of each tangent. **OR** PQ is a tangent to a circle with centre O at point P. If ΔOPQ is an isosceles triangle, then find $\angle$OQP.	1

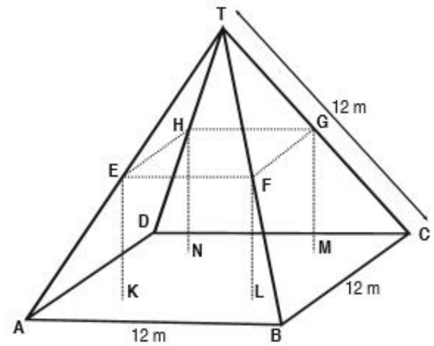
10.	In the ΔABC, D and E are points on side AB and AC respectively such that DE II BC. If AE=2cm, AD=3cm and BD=4.5cm, then find CE.	1
11.	In the figure, if B1, B2, B3,…... and A1,A2, A3,….. have been marked at equal distances. In what ratio C divides AB? 	1
12.	$Sin\ A + Cos\ B = 1,\ A = 30°$ and B is an acute angle, then find the value of B.	1
13.	If x=2sin^{2}ϴ and y=2cos^{2}ϴ+1, then find x+y	1
14.	In a circle of diameter 42cm,if an arc subtends an angle of 60˚ at the centre where ∏=22/7, then what will be the length of arc.	1
15.	12 solid spheres of the same radii are made by melting a solid metallic cylinder of base diameter 2cm and height 16cm. Find the diameter of the each sphere.	1
16.	Find the probability of getting a doublet in a throw of a pair of dice. **OR**	1

	Find the probability of getting a black queen when a card is drawn at random from a well-shuffled pack of 52 cards.	
	Section-II **Case study based questions are compulsory. Attempt any four sub parts of each question. Each subpart carries 1 mark**	
17.	**Case Study based-1** **SUN ROOM** The diagrams show the plans for a sun room. It will be built onto the wall of a house. The four walls of the sunroom are square clear glass panels. The roof is made using • Four clear glass panels, trapezium in shape, all the same size • One tinted glass panel, half a regular octagon in shape 	
(a)	**Refer to Top View** Find the mid-point of the segment joining the points J (6, 17) and I (9, 16). (i) (33/2,15/2) (ii) (3/2,1/2) (iii)(15/2,33/2) (iv) (1/2,3/2)	1

(b)	**Refer to Top View** The distance of the point P from the y-axis is (i) 4 (ii) 15 (iii) 19 (iv) 25	1
(c)	**Refer to Front View** The distance between the points A and S is (i) 4 (ii) 8 (iii)16 (iv)20	1
(d)	**Refer to Front View** Find the co-ordinates of the point which divides the line segment joining the points A and B in the ratio 1:3 internally. (i) (8.5,2.0) (ii) (2.0,9.5) (iii) (3.0,7.5) (iv) (2.0,8.5)	1
(e)	**Refer to Front View** If a point (x,y) is equidistant from the Q(9,8) and S(17,8),then (i) x+y=13 (ii) x-13=0 (iii) y-13=0 (iv)x-y=13	1
18.	**Case Study Based- 2** **SCALE FACTOR AND SIMILARITY** **SCALE FACTOR** A scale drawing of an object is the same shape as the object but a different size. The scale of a drawing is a comparison of the length used on a drawing to the length it represents. The scale is written as a ratio. **SIMILAR FIGURES** The ratio of two corresponding sides in similar figures is called the scale factor.  Scale factor $= \frac{length\ in\ image}{corresponding\ length\ in\ object}$ If one shape can become another using Resizing then the shapes are **Similar**	

	 Rotation or Turn Reflection or Flip   Translation or Slide Hence, two shapes are **Similar** when one can become the other after a resize, flip, slide or turn.	
(a)	A model of a boat is made on the scale of 1:4. The model is 120cm long. The full size of the boat has a width of 60cm. What is the width of the scale model?  (i) 20 cm (ii) 25 cm (iii) 15 cm (iv)240 cm	1

(b)	What will effect the similarity of any two polygons? (i) They are flipped horizontally (ii)They are dilated by a scale factor (iii)They are translated down (iv)They are not the mirror image of one another	1
(c)	If two similar triangles have a scale factor of a: b. Which statement regarding the two triangles is true? (i)The ratio of their perimeters is 3a : b (ii)Their altitudes have a ratio a:b (iii)Their medians have a ratio $\frac{a}{2}$: b (iv)Their angle bisectors have a ratio $a^2 : b^2$	1
(d)	The shadow of a stick 5m long is 2m. At the same time the shadow of a tree 12.5m high is  (i)3m (ii)3.5m (iii)4.5m (iv)5m	1
(e)	Below you see a student's mathematical model of a farmhouse roof with measurements. The attic floor, ABCD in the model, is a square. The beams that support the roof are the edges of a rectangular prism, EFGHKLMN. E is the middle of AT, F is the middle of BT, G is the middle of CT, and H is the middle of DT. All the edges of the pyramid in the model have length of 12 m.	1



What is the length of EF, where EF is one of the horizontal edges of the block?

(i)24m

(ii)3m

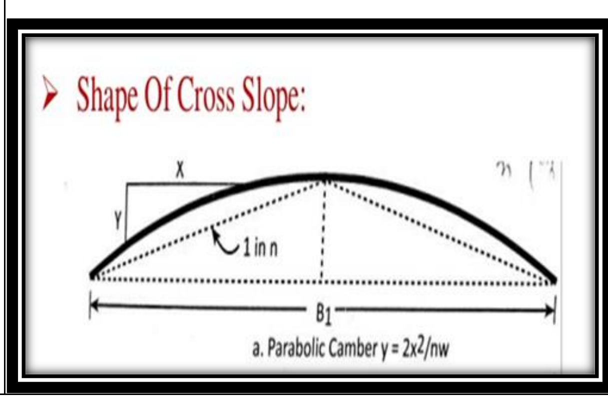
(iii)6m

(iv)10m

19. **Case Study Based- 3**

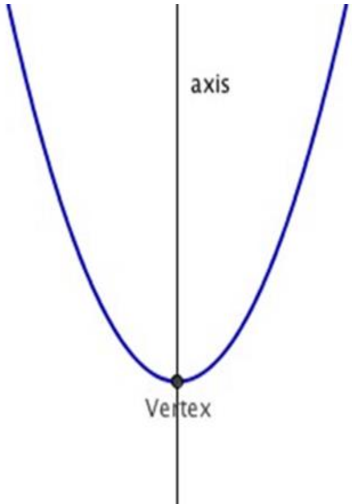
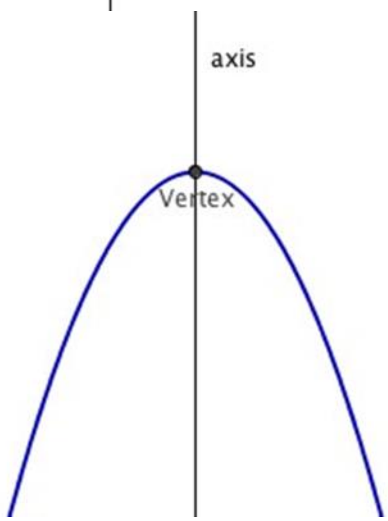
Applications of Parabolas-Highway Overpasses/Underpasses

A highway underpass is parabolic in shape.

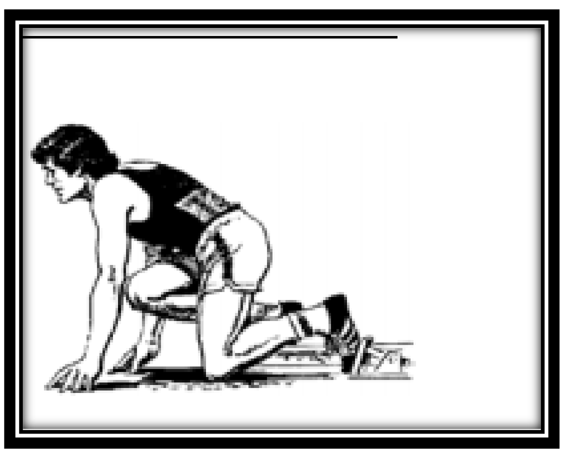


Parabola

A parabola is the graph that results from $\mathbf{p(x)}=\boldsymbol{ax^2+bx+c}$ Parabolas are symmetric about a vertical line known as the ***Axis of Symmetry***. The Axis of Symmetry runs through the maximum or minimum point of the parabola which is called the

	Vertex  	
(a)	If the highway overpass is represented by $x^2 – 2x – 8$. Then its zeroes are (i) (2,-4) (ii) (4,-2) (iii) (-2,-2) (iv) (-4,-4)	
(b)	The highway overpass is represented graphically. Zeroes of a polynomial can be expressed graphically. Number of zeroes of polynomial is equal to number of points where the graph of polynomial (i) Intersects x-axis (ii) Intersects y-axis (iii) Intersects y-axis or x-axis (iv)None of the above	

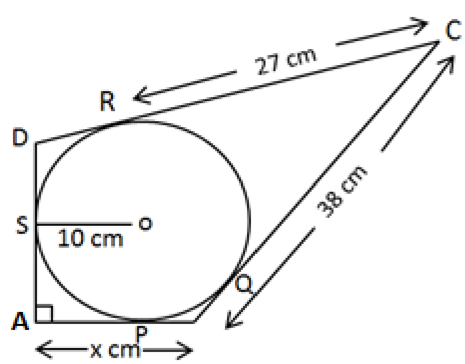
(c)	Graph of a quadratic polynomial is a (i) straight line (ii) circle (iii)parabola (iv)ellipse	
(d)	The representation of Highway Underpass whose one zero is 6 and sum of the zeroes is 0, is (i)$x^2 - 6x + 2$ (ii) $x^2 - 36$ (iii)$x^2 - 6$ (iv)$x^2 - 3$	
(e)	The number of zeroes that polynomial $f(x) = (x - 2)^2 + 4$ can have is: (i)1 (ii) 2 (iii) 0 (iv) 3	
20.	**Case Study Based- 4**	

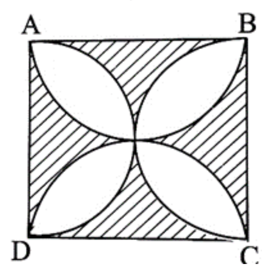


100m RACE
A stopwatch was used to find the time that it took a group of students to run 100 m.

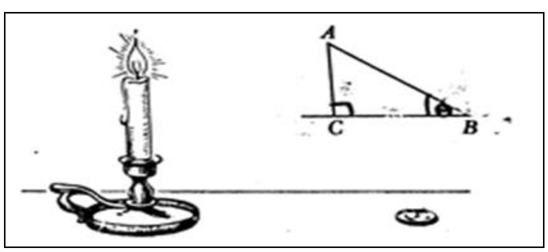
Time (in sec)	0-20	20-40	40-60	60-80	80-100
No. of students	8	10	13	6	3

(a)	Estimate the mean time taken by a student to finish the race. (i)54 (ii)63 (iii)43 (iv)50	
(b)	What wiil be the upper limit of the modal class ? (i)20 (ii)40 (iii)60 (iv)80	
(c)	The construction of cummulative frequency table is useful in determining the (i)Mean (ii)Median (iii)Mode (iv)All of the above	
(d)	The sum of lower limits of median class and modal class is (i)60 (ii)100 (iii)80 (iv)140	
(e)	How many students finished the race within 1 minute? (i)18 (ii)37 (iii)31 (iv)8	
	Part –B **All questions are compulsory. In case of internal choices, attempt any one.**	
21.	3 bells ring at an interval of 4,7 and 14 minutes. All three bell rang at 6 am, when the three balls will the ring together next?	2
22.	Find the point on x-axis which is equidistant from the points (2,-2) and (-4,2) **OR**	2

	P (-2, 5) and Q (3, 2) are two points. Find the co-ordinates of the point R on PQ such that PR=2QR	
23.	Find a quadratic polynomial whose zeroes are $5-3\sqrt{2}$ and $5+3\sqrt{2}$.	2
24.	Draw a line segment AB of length 9cm. With A and B as centres, draw circles of radius 5cm and 3cm respectively. Construct tangents to each circle from the centre of the other circle.	2
25.	If tanA=3/4, find the value of 1/sinA+1/cosA **OR** If $\sqrt{3}\sin\theta-\cos\theta=0$ and $0^\circ<\theta<90^\circ$, find the value of θ	2
26.	In the figure, quadrilateral ABCD is circumscribing a circle with centre O and AD⊥AB. If radius of incircle is 10cm, then the value of x is  	2
27..	Prove that $2-\sqrt{3}$ is irrational, given that $\sqrt{3}$ is irrational.	3
28.	If one root of the quadratic equation $3x^2+px+4=0$ is 2/3, then find the value of p and the other root of the equation. **OR** The roots α and β of the quadratic equation $x^2-5x+3(k-1)=0$ are such that $\alpha-\beta=1$. Find the value k.	3

<table>
<tr><td>29.</td><td>In the figure, ABCD is a square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region.

</td><td>3</td></tr>
<tr><td>30.</td><td>The perimeters of two similar triangles are 25cm and 15cm respectively. If one side of the first triangle is 9cm, find the length of the corresponding side of the second triangle.

OR

In an equilateral triangle ABC, D is a point on side BC such that BD = 1/3 BC. Prove that 9 $AD^2 = 7\ AB^2$</td><td>3</td></tr>
<tr><td>31.</td><td>The median of the following data is 16. Find the missing frequencies a and b, if the total of the frequencies is 70.
<table>
<tr><td>Class</td><td>0-5</td><td>5-10</td><td>10-15</td><td>15-20</td><td>20-25</td><td>25-30</td><td>30-35</td><td>35-40</td></tr>
<tr><td>Frequency</td><td>12</td><td>a</td><td>12</td><td>15</td><td>b</td><td>6</td><td>6</td><td>4</td></tr>
</table></td><td>3</td></tr>
<tr><td>32.</td><td>

If the angles of elevation of the top of the candle from two coins distant ‘a’ cm and ‘b’ cm (a>b) from its base and in the same straight line from it are 30° and 60°, then find the height of the candle.</td><td>3</td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

Section V		
33.	The mode of the following data is 67. Find the missing frequency x. <table><tr><td>Class</td><td>40-50</td><td>50-60</td><td>60-70</td><td>70-80</td><td>80-90</td></tr><tr><td>Frequency</td><td>5</td><td>x</td><td>15</td><td>12</td><td>7</td></tr></table>	3
34.	The two palm trees are of equal heights and are standing opposite each other on either side of the river, which is 80 m wide. From a point O between them on the river the angles of elevation of the top of the trees are 60° and 30°, respectively. Find the height of the trees and the distances of the point O from the trees. **OR** The angles of depression of the top and bottom of a building 50 meters high as observed from the top of a tower are 30˚ and 60˚ respectively. Find the height of the tower, and also the horizontal distance between the building and the tower.	5
35.	Water is flowing through a cylindrical pipe of internal diameter 2cm, into a cylindrical tank of base radius 40 cm at the rate of 0.7m/sec. By how much will the water rise in the tank in half an hour?	5
36.	A motorboat covers a distance of 16km upstream and 24km downstream in 6 hours. In the same time it covers a distance of 12 km upstream and 36km downstream. Find the speed of the boat in still water and that of the stream.	5

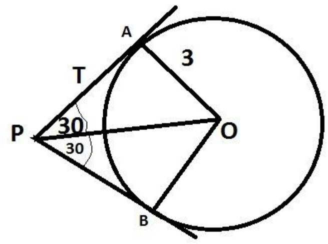
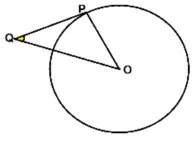
MARKING SCHEME SQP

MATHEMATICS (STANDARD)

2020-21

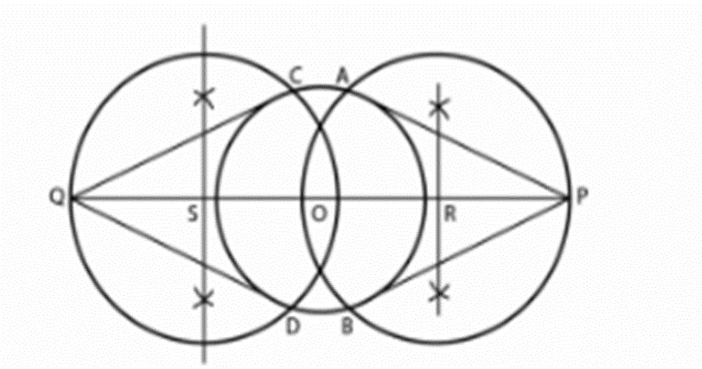
CLASS X

S.NO.	ANSWER	MARKS
	Part-A	
1.	(LCM)(3) =180 LCM=60 **OR** Four decimal places	½ ½ 1
2.	$\alpha+\beta=k/3$ 3=k/3 K=9	½ ½
3.	$\frac{3}{6}=\frac{1}{k}\neq\frac{3}{8}$ $\frac{3}{6}=\frac{1}{k}$ K=2	½ ½
4.	Let the cost of 1 chair=Rs.x And the cost of 1 table=Rs. y 3x+y=1500 6x+y=2400	½ ½
5.	$a_n=a+(n-1)d$ 0=27+(n-1)(-3) 30=3n n =10 10^{th} **OR** an=a+(n-1)d 4=a+6x(-4) a=-28	 ½ ½ ½ ½
6.	$9x^2+6kx+4=0$ $(6k)^2-4X9X4=0$ $36k^2=144$ $K^2=4$ $K=\pm2$	 ½ ½

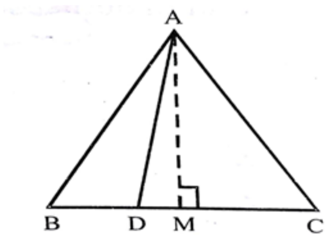
7.	$x^2+7x+10=0$ $x^2+5x+2x+10=0$ $(x+5)(x+2)=0$ X=-5, x= - 2 **OR** $3ax^2-6x+1=0$ $(-6)^2-4(3a)(1)<0$ 12a>36 =>a>3	 ½ ½ ½ ½
8.	PQ=PT PL+LQ=PM+MT PL+LN=PM+MN Perimeter(∆PLM) =PL+LM+PM =PL+LN+MN+PM =2(PL+LN) =2(PL+LQ) =2X28=56cm	 ½ ½
9.	 In ∆PAO Tan30˚=AO/PA 1/√3 =3/PA PA=3√3 cm **OR**  In ∆OPQ <P+<Q+<O=180˚ 2<Q+<P=180˚ 2<Q+90˚=180˚ 2<Q=90˚ <Q= 45˚	 ½ ½ ½ ½

10.	$\frac{AD}{BD}=\frac{AE}{CE}$ $\frac{3}{4.5}=\frac{2}{CE}$ CE=3cm	½ ½
11.	8:5	1
12.	Sin30˚+cosB=1 ½+cosB=1 CosB=1/2 B=60˚	½ ½
13.	x+y $=2\sin^2\theta + 2\cos^2\theta + 1$ $=2(\sin^2\theta + \cos^2\theta)+1$ = 3	½ ½
14.	length of arc=Ө/360˚(2∏r) = 60/360(2X22/7X21) =22 cm	½ ½
15.	$\prod R^2H=12X4/3\prod r^3$ $1X1x16=4/3Xr^3 X12$ $r^3=1$ r=1 d=2cm	 ½ ½
16.	probability of getting a doublet=1/6 **OR** probability of getting a black queen=2/52=1/26	1
17.	(a) iii)(15/2,33/2) (b) i) 4 (c) iii)16 (d) iv)(2.0,8.5) (e) ii) x-13=0	1x4=4
18.	(a) iii)15 cm (b) iv)They are not the mirror image of one another (c) ii)Their altitudes have a ratio a:b (d) iv) 5m (e) iii)6m	1x4=4
19.	(a) ii) (4,-2) (b) i) Intersects x-axis (c) iii) parabola	1x4=4

	(d)	ii) $x^2 - 36$	
	(e)	iii) 0	
20.	(a)	iii)43	1x4=4
	(b)	iii)60	
	(c)	ii)Median	
	(d)	iii)80	
	(e)	iii)31	

	Part-B	
21.	4=2X2 7=7X1 14=2X7 LCM=2X2X7=28 The three bells will ring together again at 6:28 am	½ ½ ½ ½
22.	Let P(x,0) be a point on X-axis PA=PB $PA^2=PB^2$ $(x-2)^2+(0+2)^2=(x+4)^2+(0-2)^2$ $X^2+4-4x+4=x^2+16+8x+4$ -4x+4=8x+16 X=-1 P(-1,0) **OR** PR:QR=2:1 $R\left(\frac{1(-2)+2(3)}{2+1}\ ,\ \frac{1(5)+2(2)}{2+1}\right)$ R(4/3, 3)	½ ½ ½ ½ ½ **1** ½
23.	Sum of zeroes= $5-3\sqrt{2}+5+3\sqrt{2}=10$ Product of zeroes= $(5-3\sqrt{2})(5+3\sqrt{2})= 7$ $P(x)= X^2-10x+7$	½ **1** ½
24.		Line seg=1/2 Circles=1/2 Tangents =1/2+ ½

25.	tanA=3/4=3k/4k sinA=3k/5k=3/5,cosA=4k/5k=4/5 1/sinA+1/cosA =5/3+5/4 =(20+15)/12 =35/12 **OR** √3 sinΘ=cosΘ sinΘ/cosΘ=1/√3 tanΘ=1/√3 Θ=30˚	½ ½ ½ ½ ½ ½ ½ ½
26.	<A = <OPA =<OSA = 90˚ Hence, <SOP=90˚ Also, AP=AS Hence, OSAP is a square AP=AS=10cm CR=CQ=27cm BQ=BC-CQ=38-27=11cm BP=BQ=11 cm X=AB=AP+BP=10+11=21 cm	½ ½ ½ ½
27.	Let 2-√3 be a rational number We can find co-prime a and b (b≠0) such that 2-√3=a/b 2-a/b=√3 So we get,(2a-b)/b=√3 Since a and b are integers, we get (2a-b)/b is irrational and so √3 is rational. But √3 is an irrational number Which contradicts our statement Therefore 2-√3 is irrational	½ ½ ½ ½ ½ ½
28.	$3x^2+px+4=0$ 3(2/3)2+p(2/3)+4=0 4/3+2p/3+4=0 P=-8 $3x^2-8x+4=0$ $3x^2-6x-2x+4=0$ X=2/3 or x=2 Hence, x=2	½ ½ ½ ½ ½ ½

	OR $\alpha+\beta=5$ ----(1) $\alpha-\beta=1$ ----(2) Solving (1) and (2), we get $\alpha=3$ and $\beta=2$ also $\alpha\beta=6$ or $3(k-1)=6$ $k-1=2$ $k=3$	 ½ ½ ½ ½ ½ ½
29.	Area of 1 segment = area of sector –area of triangle $=(90^\circ/360^\circ)\pi r^2 - \frac{1}{2}\times 7\times 7$ $=1/4\times 22/7\times 7^2 - \frac{1}{2}\times 7\times 7$ $= 14cm^2$ Area of 8 segments$=8\times 14= 112\ cm^2$ Area of the shaded region $= 14\times 14-112$ $=196-112=84cm^2$ *(each petal is divided into 2 segments)*	½ ½ ½ ½ ½ ½
30.	$\Delta ABC \sim \Delta DEF$ $\frac{Perimeter\ (\Delta ABC)}{Perimeter\ (\Delta DEF)}=\frac{AB+BC+CA}{DE+EF+FD}=\frac{AB}{DE}$ $\frac{25}{15}=\frac{9}{X}$ $X=5.4cm$ $DE=5.4cm$ **OR**  Construction-Draw $AM \perp BC$ $BD \perp 1/3\ BC$, $BM=1/2\ BC$ In ΔABM, $AB^2=AM^2+BM^2$ $=AM^2+(BD+BM)^2$ $=AM^2+DM^2+BD^2+2BD.\ DM$ $=AD^2+BD^2+2BD(BM-BD)$ $=AD^2+(BC/3)^2+2.\ BC/3.(BC/2-BC/3)$ $=AD^2+2BC^2/9$ $=AD^2+2AB^2/9$ Hence, $7AB^2=9AD^2$	 1 ½ ½ 1 ½ ½ ½ ½ ½ ½

31.

Class	Frequency	Cumulative frequency
0-5	12	12
5-10	a	12+a
10-15	12	24+a
15-20	15	39+a
20-25	b	39+a+b
25-30	6	45+a+b
30-35	6	51+a+b
35-40	4	55+a+b
Total	70	

1

½

55+a+b=70
a+b=15

median=l+$\frac{\frac{N}{2}-cf}{f}$ X h ½

16 =15+$\frac{35-24-a}{15}$ X 5

1=(11-a)/3

A=8

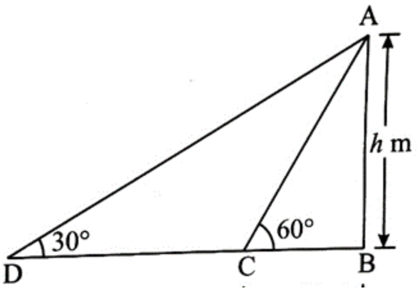
½

55+a+b=70 ½

55+8+b=70

B=7

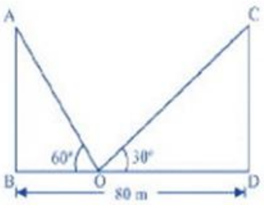
32.

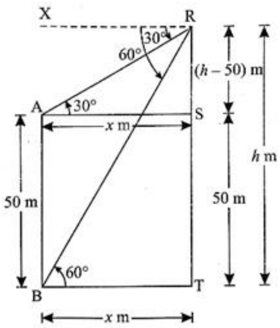


½

½

Let AB=candle
C and D are coins
Tan60˚=AB/BC=h/b
√3=h/b
H=b√3 --------------(1) ½
Tan30˚=AB/BD=h/a
1/√3=h/a
H=a/√3 --------------(2) ½
Multiplying (1) and (2), we get
H^2= b√3X a/√3 ½
H^2= b a
H=√ab m ½

33.	Mode= $l+\frac{f1-f0}{2f1-f2-f0}$ xh $67 = 60+\frac{15-x}{30-12-x}$x 10 $7 = \frac{15-x}{18-x}$x 10 7x(18-x)=10(15-x) 126-7x=150-10x 3x=150-126 3x=24 X=8	½ ½ ½ ½ ½ ½
34.	 Let BD=river AB=CD=palm trees=h BO=x OD=80-x In ∆ABO, Tan60˚=h/x √3=h/x -----------------------(1) H=√3x In ∆CDO, Tan 30˚=h/(80-x) 1/√3= h/(80-x) ---------------------(2) Solving (1) and (2), we get X=20 H=√3x=34.6 the height of the trees=h=34.6m BO=x=20m DO=80-x=80-20=60m	1 ½ ½ ½ ½ ½ ½ ½ ½

	OR  Let AB=Building of height 50m RT= tower of height= h m BT=AS=x m AB=ST=50 m RS=TR-TS=(h-50)m In ∆ARS, tan30˚=RS/AS 1/√3 = (h-50)/x -------------(1) In ∆RBT, tan60˚=RT/BT √ 3 = h/x --------------(2) Solving (1) and (2), we get h= 75 from (2) x=h/√3 =75/√3 =25√3 Hence, height of the tower=h=75m Distance between the building and the tower=25√3=43.25m	1 ½ ½ ½ ½ ½ ½ ½ ½
35.	For pipe , r = 1cm Length of water flowing in 1 sec, h=0.7m=7cm Cylindrical Tank,R=40 cm , rise in water level=H Volume of water flowing in 1 sec= ∏r^2h=∏x1x1x70 =70∏ Volume of water flowing in 60 sec=70∏x60 Volume of water flowing in 30 minutes=70∏x60x30 Volume of water in Tank=∏r^2H=∏x40x40xH Volume of water in Tank= Volume of water flowing in 30 minutes ∏x40x40xH = 70∏x60x30 H=78.75cm	½ ½ ½ ½ 1 ½ ½ ½ ½

36.	Let speed of the boat in still water =x km/hr, and Speed of the current =y km/hr Downstream speed =(x+y) km/hr Upstream speed =(x−y) km/hr $\frac{24}{x+y} + \frac{16}{x-y} = 6$--------(1) $\frac{36}{x+y} + \frac{12}{x-y} = 6$----------(2) Let $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v Put in the above equation we get, 24u+16v=6 Or, 12u+8v=3 ... (3) 36u+12v=6 Or, 6u+2v=1 ... (4) Multiplying (4) by 4, we get, 24u+8v=4v … (5) Subtracting (3) by (5), we get, 12u=1 ⇒u=1/12 Putting the value of u in (4), we get, v=1/4 ⇒ $\frac{1}{x+y} = \frac{1}{12}$ and $\frac{1}{x-y} = \frac{1}{4}$ ⇒x+y=12 and x−y=4 Thus, speed of the boat in still water = 8 km/hr, Speed of the current = 4 km/hr	½ ½ ½ ½ ½ ½ ½ ½ ½ ½

KENDRIYA VIDYALAYA SANGATHAN AHMEDABAD REGION

FIRST PRE-BOARD EXAMINATION - 2020-21

CLASS-X

MATHEMATICS - BASIC (241)

Time Allowed: **Maximum Marks: 80**

1. For Reading the Question Paper: 15 Minutes
2. For Writing Answers: 3 Hours

General Instructions:

1. This Question Paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

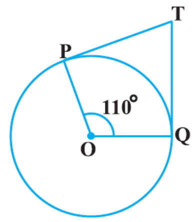
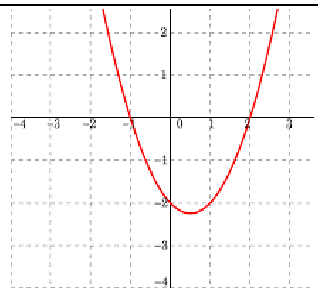
Part A

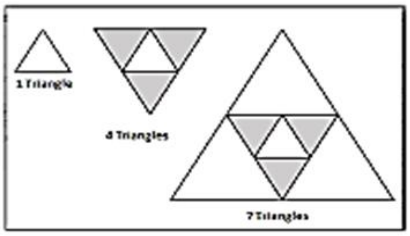
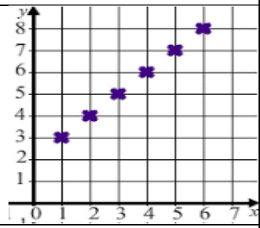
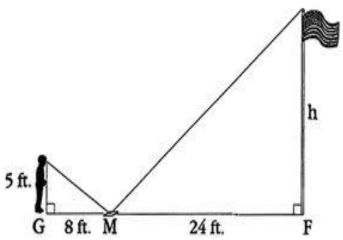
1. It consists two sections I and II.
2. Section I has 16 questions of 1 mark each. Internal choice is provided in 5 questions.
3. Section II has 4 questions on case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any four out of 5 sub-parts.

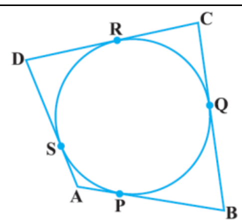
Part B

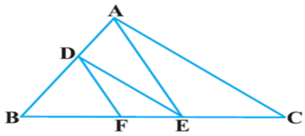
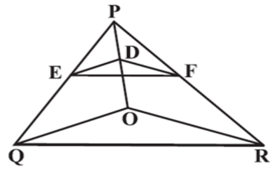
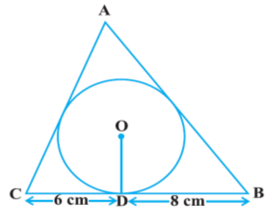
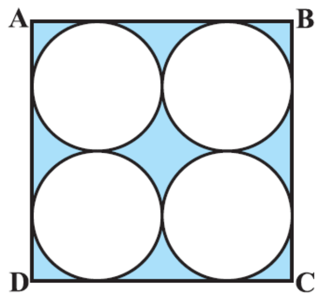
1. Question No. 21 to 26 are very short answer type questions of 2 marks each.
2. Question No. 27 to 33 are short answer type questions of 3 marks each.
3. Question No. 34 to 36 are long answer type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1question of 5 marks.

Q.NO		Part-A (Section-I) (1-mark each)	MM
1		Write the smallest number that is divisible by all the numbers from 1 to 5 (both inclusive). **OR** Express 135 as the product of primes numbers.	1
2		Write a quadratic polynomial, sum and product of whose zeroes are -3 and 4 Respectively.	1
3		On comparing the ratio of the coefficients, find out whether the pair of linear equations $x + 3y = 11$ and $4x + 12y = 22$ is Consistent or Inconsistent.	1
4		What will be the value of k If the lines given by 3x + 2ky = 12 and 2x + 4y = 8 are parallel.	1
5		Find the roots for quadratic equation: $x^2 – 4 = 0$	1
6		Find the values of k for which the quadratic equation $2x^2 – kx + k = 0$ has equal roots. **OR** The area of a rectangular sand art is 10 square metre. The length of the sand art (in metres) is one more than twice its breadth. Write the quadratic equation for the given condition.	1
7		Write the 10^{th} term of the AP: 5, 8, 11, 14, ……….. **OR** If the common difference of an AP is 5, then what is the value of $a_{18} – a_{13}$	1

8		Find the perpendicular distance of the point P (2, 3) from the *x*-axis.	1
9		The value of (sin30° + cos30°) – (sin60° + cos60°)	1
10		If Δ ABC is right angled at C, then find the value of COS (A+B). **OR** For SinA = CosA find the value of angle A where $0^0 < A < 90^0$	1
11		The lengths of the diagonals of a rhombus are 16 cm and 12 cm. Then, what will be the length of the side of the rhombus.	1
12		A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Find the Length PQ.	1
13		If TP and TQ are the two tangents to a circle with centre O so that∠ POQ = 110°,then find the value of ∠ PTQ.  **OR** From an exterior point T, the length of the tangent TQ to a circle is 24 cm and the distance of T from the centre O is 25 cm. Find the radius of the circle.	1
14		To divide a line segment AB in the ratio 5:7, a ray AX is drawn so that ∠ BAX is an acute angle and then at equal distances points are marked on the ray AX what will be the minimum number of these points marked.	1
15		If the area of a circle is 154 cm^2 , then find its circumference.	1
16		If the circumference of a circle is equal to perimeter of a square, then find the area of square in term of π and radius r.	1
Part-A Section- II, (1-mark each) **Case study –based questions are compulsory. Attempt any 4 sub parts from each questions. Each question carries (1 mark)**			
17		Raju has seen that his uncle is working on computer, he imitates the shape from computer on a graph paper.  **Answer the following questions below.**	
	1	Name the shape Raju's uncle drawn. a) Spiral b) Ellipse c) Linear d) Parabola	1
	2	Degree of quadratic polynomial is a) 3 b) 4 c) 2 d) 1	1
	3	5 is an example of a) Zero polynomial b) constant polynomial c) linear polynomial d) variable	1
	4	The shape drawn on graph may represent a) –x – 2 b) y = - x +2 c) y= $x^2 - x$ -2 d) y = x	1
	5	The number of zeroes of the polynomial (shape drawn) a) 2 b) 3 c) 4 d) 0	1

18		A painter is painting triangular designs on a stadium wall is as shown in adjoining figure.  **Answer the following questions given below.**	
	1	If we treat these designs as an AP (beginning from left most design),what will be number of triangles in the fourth design . a) 9 b) 10 c) 11 d) 12	1
	2	What is the common difference , in above pattern of designs . a) 3 b) 4 c) 5 d) 6	1
	3	If he continue to paint with this pattern , what will be the number of total triangles up to sixth design . a) 51 b) 52 c) 53 d) none of these.	1
	4	What will be number (order) of design (term)in which19 triangles will be made. a) 8^{th} term b) 7^{th} term c) 9^{th} term d) none of these	1
	5	In any AP, difference between two consecutive terms are. a) Equal b) increasing c) decreasing d) zero	1
19		Rajni planted some Neem tree sapling in her garden . The graphical representation of the situation is shown in adjoining figure.  Based on this , Answer the questions given below.	
	1	Find the distance between first (1,3) and last (6,8) sapling planted (in units). a) $5\sqrt{2}$ units b) 4 units c) 5 units d) none of these	1
	2	Which one is not a possible coordinate for any of the sapling planted . a) (1,3) b) (6,8) c) (2,4) d) (3,1)	1
	3	What is the coordinate of the mid point between first (1,3) and last (6,8) sapling planted. a) (7/2 ,11/2) b) (3,5) c) (2,3) d) (0,0)	1
	4	The coordinates of the points ,where the distance between first (1,3) and last (6,8) sapling planted ,can be divided in 1:4 . a) (1,3) b) (6,8) c) (2,4) d) (3,1)	1
	5	The X coordinate is also known as a) Grid b) Ordinate c) Abscissa d) Co-ordinate	1
20		There is a mirror M situated on the ground. Roshan is watching the reflection of the flag in the mirror. This condition is shown with the help of two triangles in the adjoining figure.If point P represents head of Roshan and point Q represents top of the pole (in which the flag is firling). We know with (The help of law of reflection) ∠ PMG = ∠QMF.  Answer the questions given below.	
	1	Δ PGM is similar to a) Δ FMQ b) Δ QMF c) Δ MQF d) Δ QFM	1

	2	The two triangles are similar by , which criterion of similarity. a) AA b) SSS c) QMF d) PMQ	1
	3	The height of pole (the value of h) is a) 10 ft b) 20 ft c) 15 ft d) none of these	1
	4	In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.” This statement is known as a) Thales theorem b) Pythagoras theorem c) similarity theorem d) none of these	1
	5	What is the ratio ,you found between two corresponding sides in the condition given above a) 5:24 b) 1:3 c) 12:5 d) 3:5	1
Part-B (Section-I) (2 marks each) **All questions are compulsory. In case of internal choices, attempt any one.**			
21		Find the HCF of 96 and 404 by the prime factorisation method. also find their LCM.	2
22		Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify The relationship between the zeroes and the coefficients.	2
23		Find the point on the *x*-axis which is equidistant from (2, –5) and (–2, 9). **OR** Find the values of *y* for which the distance between the points P(2, -3) and Q(10, *y*) is 10 units.	2
24		Given 15cot A = 8, find sinA and secA. **OR** If cos A $=\frac{12}{13}$, find sinA and tanA	2
25		A quadrilateral ABCD is drawn to circumscribe a circle . Prove that AB + CD = AD+BC. 	2
26		Draw a line segment of length 10 cm and divide it in the ratio 3:2 . Measure the two parts.	2
Part-B (Section-II) (3 marks each) **All questions are compulsory. In case of internal choices, attempt any one**			
27		Prove that $\sqrt{\mathbf{2}}$ is an irrational number.	3
28		The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, Find the other two sides. **OR** Find the values of *k* for each of the following quadratic equations, so that they have two equal roots : $\boldsymbol{kx(x-2)+6=0}$	3
29		Prove that $(\mathbf{cosec\ A - cot\ A})^2 = \frac{1-COSA}{1+COSA}$	3
30		An observer 1.5 m tall is 28.5 m away from a chimney. The angle of elevation of the top of the chimney from her eyes is 45°. What is the height of the chimney?	3
31		In fig (1) DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$ (see figure adjoined)	3

		 Fig(1) OR  Fig(2) In Fig (2) DE || OQ and DF || OR. Show that EF || QR.	
32		A Δ ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8cm and 6cm respectively. Find the sides AB and AC.  	3
33		Find the area of a quadrant of a circle whose circumference is 22 cm	3
Part-B (Section-III) (5 marks each) **All questions are compulsory. In case of internal choices, attempt any one**			
34		The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?	5
35		A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, The angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal. **OR** The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.	5
36		Find the area of the shaded region in adjoining figure, where ABCD is a square of side 14 cm.  	5

MARKING SCHEME

KENDRIYA VIDYALAYA SANGATHAN AHMEDABAD REGION

FIRST PRE-BOARD EXAMINATION - 2020-21

CLASS-X

MATHEMATICS - BASIC (241)

QUE		PART-A (SECTION-I)	MM
1		2 x 2 x 3 x 5 = 60 OR 3^3 x 5	1
2		$x^2 + 3x + 4$	1
3		Inconsistent	1
4		The value of k = 3	1
5		x = 2 and x = -2	1
6		The values 0 or 8 OR $2x^2 + x - 10 = 0$	1
7		The 10^{th} term of the AP: 5, 8, 11, 14, ... is 32 OR The value = 25	1
8		The distance of the point P (2, 3) from the x-axis = 3 units	1
9		The value of (sin30° + cos30°) – (sin60° + cos60°) = 0	1
10		If triangle ABC is right angled at C, then the value of cos (A+B) = 0	1
11		The length of the side of the rhombus = 10 cm	1
12		The Length PQ = $\sqrt{119}$ cm	1
13		The value of ∠ PTQ =70^0 OR The radius of the circle =7cm	1
14		The minimum number of these points = 12	1
15		Circumference = 44 cm	1
16		Area of square in term of π and radius-r = $\frac{\pi^2 r^2}{4}$	1

PART-A (SECTION-II, 1-MARK EACH)

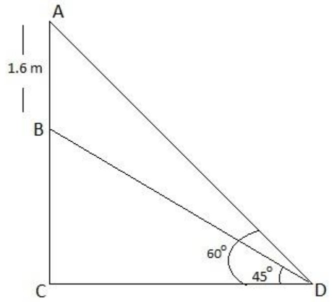
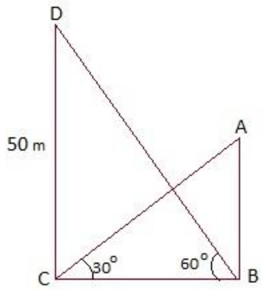
		(I)	(II)	(III)	(IV)	(V)	1-MARK EACH
17		d) Parabola	c) 2	b) Constant Polynomial	c) $y = x^2 - x - 2$	a) 2	**4**
18		b) 10	a) 3	a) 51	b) 7^{th} term	a) Equal	**4**
19		$5\sqrt{2}$ units	d) (3,1)	a) $(\frac{7}{2}, \frac{11}{2})$	c) (2,4)	d) Abscissa	**4**
20		d) Δ QFM	a) AA	c) 15 ft	b) Pythagoras theorem	b) 1:3	**4**

PART-B (SECTION-I , 2-MARKS EACH)

			MM
21		96 = 2 x 2 x 2 x 2 x 2 x 3 404 = 2 x 2 x 101 HCF =4 , LCM = 9696	1 1
22		$x^2 + 7x + 10 = (x + 2)(x + 5)$ So, the value of $x^2 + 7x + 10$ is zero when $x + 2 = 0$ or $x + 5 = 0$, i.e., when $x = -2$ or $x = -5$. Therefore, the zeroes of $x^2 + 7x + 10$ are – 2 and – 5. Now,verification sum of zeroes = -2+(-5) = $\frac{-7}{1} = \frac{-b}{a}$ and product of zeroes= 10 x 1 = $\frac{10}{1} = \frac{c}{a}$	1 1
23		Distance formula Using the formula and finding (-7, 0) OR values of $y = 3$ OR $y = -9$,using distance formula and equations	½ 1 ½
24		sin A = $\frac{15}{17}$ and sec A. = $\frac{17}{8}$ OR sinA = $\frac{5}{13}$ and tanA = $\frac{5}{12}$	1 1

25		CORRECT PROOF OF AB + CD = AD+ BC (i) DR = DS (ii) BP = BQ (iii) AP = AS (iv) CR = CQ Since they are tangents on the circle from points D, B, A, and C respectively. using theorem 10.2 adding the LHS and RHS of the above equations we get, rearranging them we get, (DR+CR) + (BP+AP) = (CQ+BQ) + (DS+AS) By simplifying, AD+BC= CD+AB	1 1
26		For Correct construction ,parts will be of 6 and 4 cm	2
		PART-B (SECTION-II, 3-MARKS EACH)	
27		Proof : Let us assume, to the contrary, that $\sqrt{2}$ is rational. So, we can find integers r and s ($\neq 0$) such that $\sqrt{2} = \frac{r}{s}$. Suppose r and s have a common factor other than 1. Then, we divide by the common factor to get $\sqrt{2} = \frac{a}{b}$, where a and b are coprime. So, b $\sqrt{2}$ = a. Squaring on both sides and rearranging, we get $2b^2 = a^2$. Therefore, 2 divides a^2 . Now, by Theorem 1.3, it follows that 2 divides a. So, we can write a = 2c for some integer c. Substituting for a, we get $2b^2 = 4c^2$, that is, $b^2 = 2c^2$. This means that 2 divides b^2 , and so 2 divides b (again using Theorem 1.3 with p = 2). Therefore, a and b have at least 2 as a common factor. But this contradicts the fact that a and b have no common factors other than 1. This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational. So, we conclude that $\sqrt{2}$ is irrational.	1 1 1
28		Framing and solving equations using Pythagoras theorem Finding base =12cm ,altitude = 5cm OR K = 6 by using Discriminant = 0	2 1
29		Proving **(cosec A – cot A)^2** = $\frac{1-COS\,A}{1+COSA}$ by application LHS **(cosec A – cot A)^2** **= (cosec A - cot A)^2** $= \left(\frac{1}{sinA} - \frac{cosA}{sinA}\right)^2$ $= \left(\frac{1-cosA}{sinA}\right)^2$ $= \frac{(1-cosA)^2}{sin^2A}$ $= \frac{(1-cosA)^2}{1-cos^2A}$ $= \frac{(1-cosA)(1-cosA)}{(1-cosA)(1+cosA)}$ $= \frac{(1-cosA)}{(1+cosA)}$	1 1 1

30		writing and applying correct trigonometric ratio Correct calculation (Solving and finding) Correct answer the height of the chimney = 30 m	1 1 1
31		Proof of $\frac{BF}{FE} = \frac{BE}{EC}$ by using basic proportionality theorem correctly OR Proof of EF || QR by using basic proportionality theorem For given and To prove For Correct proof For correct answer	1 1 1
32		Calculating area of triangle .using herons formula ,AB=15 cm and AC= 13 cm For using herons formula For correct calculation For correct answer	1 1 1
33		The area of quadrant = $\frac{77}{8}$ square cm by correct application of formula Given: Cercumference = 2πr = 22 $2 \times \frac{22}{7} \times r = 22$ $r = \frac{22 \ X \ 7}{22 \ X \ 2}$ $r = \frac{7}{2}$ cm we know that for quadrant of circle , $\theta = 90^0$ Area of quadrant = $\frac{\theta}{360^0} X \pi r^2$ Area of quadrant = $\frac{90^0}{360^0} X \frac{22}{7} X \frac{7}{2} X \frac{7}{2}$ Area of quadrant = $\frac{77}{8}$ cm^2	1 1 1
PART-B (SECTION-III, 5-MARKS EACH)			
34		Let the ten's and the unit's digits in the first number be *x* and *y*, respectively. So, the first number may be written as 10 *x* + *y* in the expanded form When the digits are reversed, 10*y* + *x* According to the given condition. $(10x + y) + (10y + x) = 66$ i.e., $11(x + y) = 66$ i.e., $x + y = 6$................................ (1) We are also given that the digits differ by 2, therefore, either $x - y = 2$ ………………………..(2) or $y - x = 2$ ……………………………..(3) If $x - y = 2$, then solving (1) and (2) by elimination, we get $x = 4$ and $y = 2$. In this case, we get the number 42. If $y - x = 2$, then solving (1) and (3) by elimination, we get $x = 2$ and $y = 4$. In this case, we get the number 24. Thus, there are two such numbers 42 and 24.	2 2 1
35		Let AB be the height of statue. D is the point on the ground from where the elevation is taken. To Find: Height of pedestal = BC = AC-AB	

		 From figure, In right triangle BCD, tan 45° = BC/CD BC=CD Again, In right ΔACD, tan 60° = AC/AD BC=1.6 /(√3-1) m. Thus, the height of the pedestal is 0.8(√3+1) m. OR Let CD be the height of the tower. AB be the height of the building. BC be the distance between the foot of the building and the tower. Elevation 30 degree and 60 degree (from the tower and the building respectively).  In right ΔBCD, tan 60° = CD/BC BC= 50/√3 Again, In right ΔABC, tan 30° = AB/BC Thus, the height of the building is 50/3 m. the height of the building =$16\frac{2}{3}$m	**2** **2** **1**
36		Area of square ABCD =196 sq cm Diameter of each circle =14 cm Radius =7 cm the area of the shaded region= area of square – area of all circles = 196 -154 =42 cm^2	2 2 1

KENDRIYA VIDYALAYA SANGATHAN

Ahmedabad Region

I Pre-Board Examination 20-21

Class X Mathematics

Time Allowed: **Maximum Marks: 80**

1. for Reading the Question Paper: 15 Minutes
2. for Writing Answers: 3 Hours

General Instructions:

1. This Question Paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

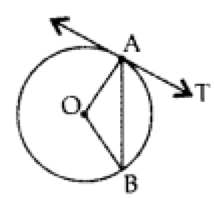
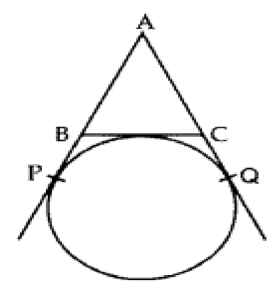
Part A

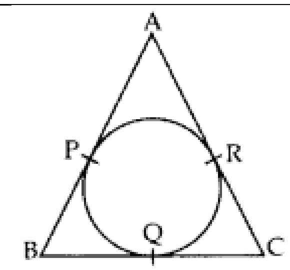
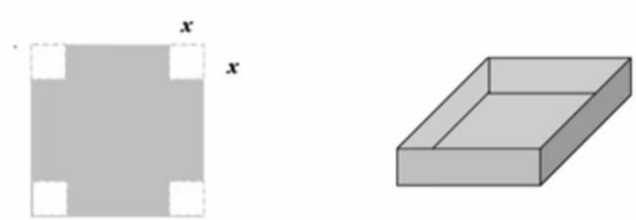
1. It consists two sections I and II.
2. Section I has 16 questions of 1 mark each. Internal choice is provided in 5 questions.
3. Section II has 4 questions on case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part B

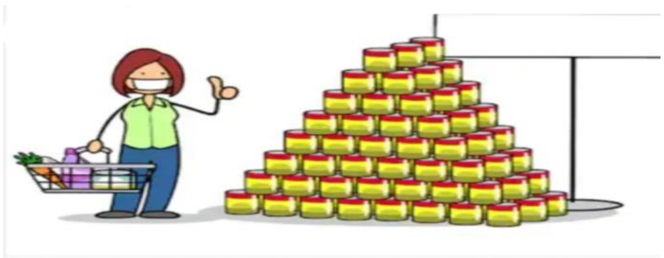
1. Question No. 21 to 26 are very short answer type questions of 2 marks each.
2. Question No. 27 to 33 are Short answer Type questions of 3 marks each.
3. Question No. 34 to 36 are long answer type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

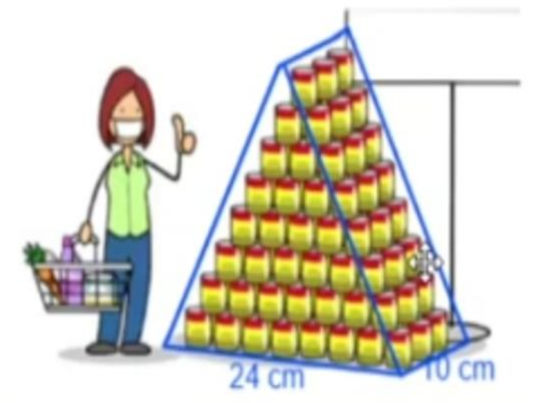
Question No.	Section-I **Section I has 16 questions of 1 mark each. Internal choice is provided in 5 questions.**	Marks allocated
1.	The HCF of two numbers is 18 and their product is 12960. Find their LCM. **OR** Without actual division, show that the following rational numbers is a terminating decimal. Express in decimal form. $\frac{19}{3125}$	**1**
2.	Find the zeroes of the polynomial $x^2 - \sqrt{2}x - 12$.	**1**
3.	Find the value of k for which the system of equations has a unique solution: $2x + 3y = 5, \quad kx - 6y = 8$.	**1**
4.	A lady has only 50-paisa coins and 25-paisa coins in her purse. If she has 50 coins in all totaling Rs.19.50, how many coins of each kind does she have? Form linear equations to represent this situation.	**1**
5.	What will be the nature of roots of quadratic equation $2x^2 + 4x - 7 = 0$	**1**
6.	For what values of k, the roots of the equation $x^2 + 4x + k = 0$ are real and equal? **OR** Find the value of k for which the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other.	**1**
7.	Find the 15th term of the AP -40,-15, 10, 35 … **OR** If the nth term of an AP is $(4n - 10)$. Find its common difference.	**1**
8.	In which quadrant the point P that divides the line segment joining the points A (2, 5) and B (5,-2) in the ratio 2:3 lies? **OR**	**1**

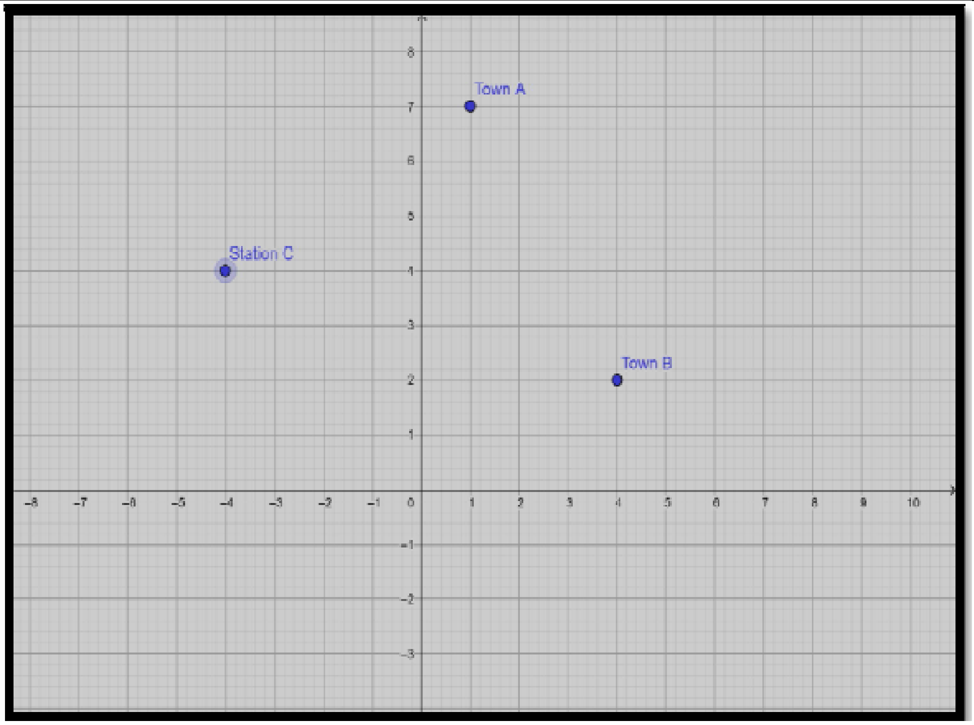
	If (2, p) is the midpoint of the line segment joining the points A (6, -5) and B (-2, 11) find the value of p.	
9.	Evaluate the following: $\sin^2 30° - \cos^2 45° + \tan^2 60°$	**1**
10.	$\tan\theta = \frac{3}{4}$,find the value of $\cos\theta + \frac{\sin\theta}{\cos\theta} - \sin\theta$	**1**
11.	In a triangle ABC, D and E are points on the sides AB and AC respectively such that DE // BC, If $\frac{AD}{DB} = \frac{2}{3}$ and AC = 18 cm, find AE.	**1**
12.	In the given figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If ∠AOB = 100°, then calculate ∠BAT. 	**1**
13.	In the given figure, AP, AQ and BC are tangents to the circle. If AB = 5 cm, AC = 6 cm and BC = 4 cm, then calculate the length of AP (in cm).  **OR**	**1**

	In the given figure, the sides AB, BC and CA of a triangle ABC touch a circle at P, Q and R respectively. If PA = 4 cm, BP = 3 cm and AC = 11 cm, find the length of BC (in cm). 	
14.	If a line-segment AB of length 7.8 cm is divided in ratio 5:8 at point P. What will be actual length of PB?	**1**
15.	The circumference of a circle is 22 cm. Calculate the area of its quadrant (in cm^2)	**1**
16.	If π = 22/7, calculate the distance (in meters) covered by a wheel of diameter 35 cm, in one revolution.	**1**
	Section-II **Case study based questions are compulsory. Attempt any four subparts of each question. Each subpart carries 1 mark**	
17.	**Case Study based-1** A box is created from a sheet of cardboard 25 inch on a side by cutting a square from each corner and folding up the sides .Let x represent the length of the sides of the squares removed from each corner .If area of the sides of the box is 13 square inch, give the answer of the following questions. 	**4**

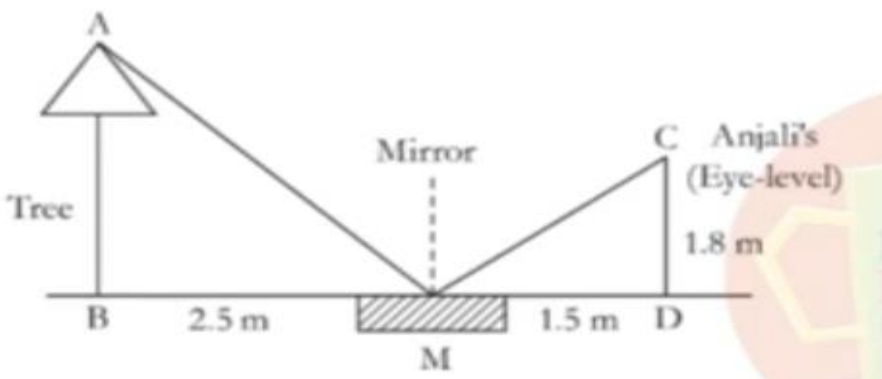
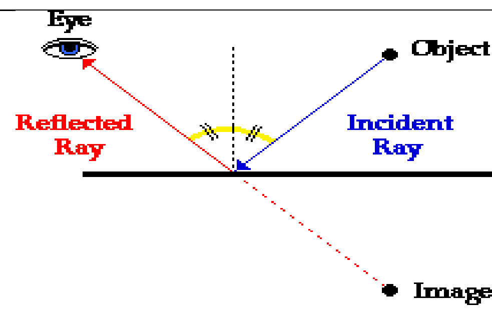
(i)	the volume of the box can be expressed by (a) x^2 X 25 (b) x^3 (c)$4x^3 + 100x^2 + 625$ (d) $4x^3 - 100x^2 + 625x$	
(ii)	If area of the sides (walls) of the box are 12 square inch then value of x is (a) 13 inch (b) 1 inch (c)0.5 inch (d) 24 inch	
(iii)	the area of the bottom of the box is (a)625 sq. Inch (b)576 sq. Inch (c)529 sq. Inch (d) 169 sq. Inch	
(iv)	the volume of the box is (a) 24 X 24 X 0.5 (b)24 X 24 X 1 (c) 13 X 13 X 12 (d)12 X !2 X 13	
(v)	The graph of the volume represented, will intersect the X-axis in point(s) (a) Zero (b)Two (c)One (d) Three	

<table>
<tr><td></td><td></td><td></td></tr>
<tr><td>18.</td><td>Case Study based-2
Rashi goes to grocery shop for purchasing some glass for gifting in a party. She observed the jars are arranged one above the other in specific pattern. Observe the figure and on the bases of the figure give answers of the following questions.
</td><td>4</td></tr>
<tr><td>(i)</td><td>If the arrangement of jars in numbers form the Arithmetic Progression, then total number of jars in first three rows from the top are _____
(a) 18
(b) 24
(c) 57
(d) 35</td><td></td></tr>
<tr><td>(ii)</td><td>If there are hundred such rows . then how many jars will be in the 56^{th} row
(a) 200
(b) 168
(c) 300
(d) 303</td><td></td></tr>
<tr><td>(iii)</td><td colspan="2">If on the top, the shopkeeper puts two more rows having jars 2 and 1 respectively, will it be an arithmetic sequence?
(a) Yes, the common difference between each row is the same.
(b) No, the common difference between each row is not the same
(c) Yes, because the common difference between each row is not same
(d) No, because the common difference between each row is the same</td></tr>
</table>

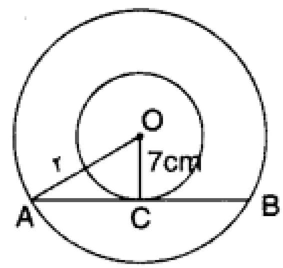
<table>
<tr><td></td><td colspan="2"></td></tr>
<tr><td>(iv)</td><td>Rashi asked the shopkeeper to pack it in the same fashion as it was displayed shopkeeper used a box of dimensions as shown. Front face of the box is an equilateral triangle, the capacity of the box used _________
(a) 480 X√3 cubic cm
(b) 240 X√3 cubic cm
(c) 2880 X√3 cubic cm
(d) 1440 X√3 cubic cm</td><td>
</td></tr>
<tr><td>(v)</td><td>Rashi asked the shopkeeper to wrap it with gift paper, the total surface area of the paper used ______
(a) √3 X 12 X 24 + 30 X 24 Sq.cm
(b) 3.14X 12 X 24 + 30 X 24 Sq.cm
(c) 1.41 X 12 X 24 + 30 X 24 Sq.cm
(d) √3 X 24 X 24 + 30 X 24 Sq.cm</td><td></td></tr>
<tr><td>19.</td><td>Case Study based-3
Two friends Seema and Aditya work in the same office at Delhi. In the Christmas vacations, both decided to go to their hometowns represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains
from the same station C (in the given figure) in Delhi. Based on the given situation, answer the following questions:</td><td>4</td></tr>
</table>

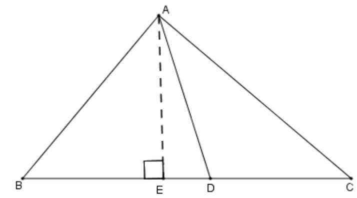
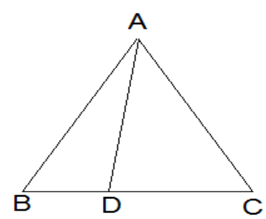
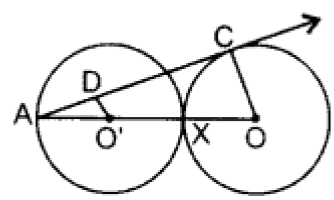
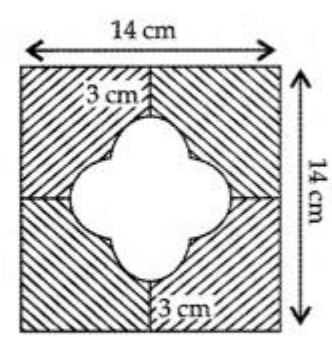
		
(i)	Which statement is true (a) Seema travel more distance than Aditya to reach to their hometown. (b) Seema travel less distance than Aditya to reach to their hometown. (c) seema and Aditya travel equal distance to reach to their hometown. (d) seema travels doubble distance than that of by aditya	
(ii)	Seema and Aditya planned to meet at a location D which is middle of Town A and Town B. D is mid-point of AB. Coordinate of point D is (a) (5,9) (b) (- 5, - 9) (c) (2.5, 4.5) (d) (2, 4)	

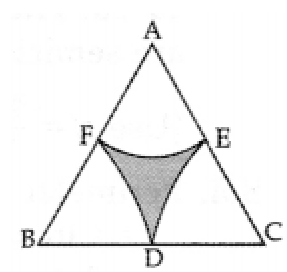
(iii)	What type of triangle is formed by joining the points represented by A, B and C. (a) Scalene triangle (b) Obtuse angled triangle. (c) Isosceles right angled triangle. (d) None of the above	
(iv)	What is the distance of station C from the origin (a) 4 unit (b) $4\sqrt{2}$ unit (c) 4.2 unit (d) 16 unit	
(v)	The area of the triangle formed by joining the points represented by A, B and C. (a) 17 sq. Unit (b) $\sqrt{68}$ sq. Unit (c) 34 sq. Unit (d) 10 sq. Unit	
20.	**Case Study based-4** Teacher gives an activity to the students to measure the height of the tree and ask them who will do this activity. Anju accepts the challenges .she places a mirror on level ground to determine the height of a tree. She stands at a certain distance so that she can see the top of the tree reflected from the mirror. Anju's eye level is 1.8 m above ground, the distance of Anju and the tree from the mirror are 1.5 m and 2.5 m respectively. Answer the question below	**4**

	 	
(i)	Name the two similar triangles formed (a) △ABM~ △CMD (b) △AMB~ △CDM (c) △ABM~△CDM (d) None of these	
(ii)	State the criteria of similarity that is applicable here (a) SSS criterion (b) SAS criterion (c) AA criterion (d) ASA criterion	
(iii)	.the height of the tree is (a) 3 m (b) 3.5 m (c) 2.5 m (d) 4 m	

(iv)	If △AMB and △CDM are similar and CD= 6 cm , MD = 8 cm , and BM = 24 cm then AB = ? (a) 17 cm (b) 18 cm (c) 12 cm (d) 24 cm	
(v)	In △AMB if ∠BAM = 30^0 ,then ∠MCD = ? (a) 40^0 (b) 45^0 (c) 60^0 (d) 30^0	
	Part B **All questions are compulsory. In case of internal choices, attempt any one.**	
	Section III	
21.	Three measuring rods are 64 cm, 80 cm and 96 cm in length. Find the least length of cloth that can be measured an exact number of times, using any of the rods.	**2**
22.	If the sum of the zeroes of the quadratic polynomial $x^2 + 2x + 3k$ is equal to the product of its zeroes, then k =?	**2**
23.	In what ratio is the line segment joining the points A (-2, -3) and B(3,7) divided by the y- axis? Also, find the coordinates of the point of division. **OR** Show that the points A(3,1),B(0,- 2),C(1,1)and D(4,4)are the vertices of parallelogram ABCD.	**2**
24.	If (1 + cos A) (1 – cos A) =3/4, find the value of sec A.	

	OR Solve the equation for θ: $\cos^2\theta / (\cot^2\theta - \cos^2\theta) = 3$	
25.	Two concentric circles are of radii 7 cm and r cm respectively, where r > 7. A chord of the larger circle, of length 48 cm, touches the smaller circle. Find the value of r. 	**2**
26.	Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.	**2**
	Section IV	
27.	Prove that $\sqrt{5}$ is irrational number.	**3**
28.	The speed of a boat in still water is 8 km/hr. It can go 15 km upstream and 22 km downstream is 5 hours. Find the speed of the stream. **OR** In a class test, the sum of the marks obtained by P in mathematics and science is 28. Had he got 3 more marks in mathematics and 4 marks less in science, the product of marks obtained in the two subjects would have been 180. Find the marks obtained by him in the two subjects separately.	**3**
29.	Prove that: $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$.	**3**
30.	The length of a string between a kite and a point on the ground is 90 metres. If the string makes an angle with the ground level such that $\tan \emptyset = 15/8$, how high is the kite? Assume that there is no slack in the string.	**3**

31.	ΔABC, if AD is the median, then show that $AB^2+AC^2 = 2(AD^2+BD^2)$ **OR**		**3**
	In an equilateral triangle ABC, D is a point on the side BC such that BD=BC/3 Prove that $9AD^2 = 7AB^2$		
32.	In the figure, two equal circles, with centres O and O', touch each other A at X. OO' produced meets the circle with centre O' at A. AC is tangent to the circle with centre O, at the point C. O'D is perpendicular to AC. Find the value of DO'/ CO.		**3**
33.	In Figure, find the area of the shaded region. [Use π = 3.14]		**3**

	Section V		
34.	2 men and 5 boys can finish a piece of work in 4 days, while 3 men and 6 boys can finish it in 3 days. Find the time taken by one man alone to finish the work and that taken by one boy alone to finish the work.		**5**
35.	A parachutist is descending vertically and makes angles of elevation of 45° and 60° at two observing points 100 m apart from each other on the left side of himself. Find the maximum height from which he falls and the distance of the point where he falls on the ground from the just observation point. **OR** A man sitting at a height of 20 m on a tall tree on a small island in the middle of a river observes two poles directly opposite to each other on the two banks of the river and in line with the foot of tree. If the angles of depression of the feet of the poles from a point at which the man is sitting on the tree on either side of the river are 60° and 30° respectively. Find the width of the river.		**5**
36.	In Figure, arcs are drawn by taking vertices A, B and C of an equilateral triangle ABC of side 14 cm as centres to intersect the sides BC, CA and AB at BZ their respective mid-points D, E and F. Find the area of the shaded region. [Use π = 22/7 and √3 = 1.73]		**5**

KENDRIYA VIDYALAYA SANGATHAN

Ahmedabad Region

I Pre-Board Examination 20-21

Mathematics

Class X

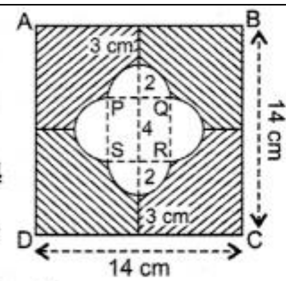
ANSWER KEY

Question No.		Marks
1.	LCM is 720. **OR** 0.00608	1
2.	$x = 3\sqrt{2}$ or $x = -2\sqrt{2}$	1
3.	$k \neq -4$, all other value except -4	1
4.	$x + y = 50$ ....(i) $0.5x + 0.25y = 19.50$ ....(ii)	1
5.	Real and unequal	1
6.	k=4 OR k=3	1
7.	15th term of the AP =310 OR Common difference = 4	1
8.	IV quadrant P (16/5, 11/5) OR the value of p = 3.	1
9.	-11/4	1
10.	value = 19/20	1
11.	7.2 cm	1
12.	$\angle BAT = 50°$	1
13.	AP = 7.5 cm OR BC = BQ + QC= 3 + 7 = 10 cm	1
14.	4.8 cm	1
15.	77/8 sq.cm	1
16.	distance = Perimeter = $2\pi r = 2 \times 22/7 \times 35/2$ cm = 110 cm or 1.1 m	1
17.	i. (d) $4x^3 - 100x^2 + 625x$ ii. (c)0.5 inch iii. (b)576 sq. Inch iv. (a) 24 X 24 X 0.5 v. (d) Three	1 1 1 1 1

18.	i.(a)18 ii.(b)168 iii.(b) No, the common difference between each row is not the same iv.(d)1440 X√3 cubic cm v.(a)√3 X 12 X 24 + 30 X 24 Sq.cm	1 1 1 1 1 1
19	i.(b)Seema travel less distance than Aditya to reach to their hometown ii.(c)(2.5, 4.5) iii. (c) isosceles right angled triangle. iv.(b)4√2 unit v. (a)17 sq. Unit	1 1 1 1 1
20	i.(c)△ABM~△CDM ii.(c)AA criterion iii.(d)3 m iv. (b)18 cm v.(d)30^0	1 1 1 1 1
21	LCM = product of greatest power of each prime factor involved in the numbers 26 × 3 × 5 = 960cm = 9.6m Hence, the required length of cloth is 9.6m.	1 1
22	$k \Rightarrow \alpha+\beta=\alpha\beta$ ⇒ −2 = 3 k ⇒ k = −2 / 3	1 ½ 1/2
23	.P lies on the y-axis; so, its abscissa is 0. the x-axis divides the line AB in the ratio 2 : 3 at the point P. the point of intersection of AB and the x-axis is P(0,1). OR	1 1

	We know that the diagonals of a parallelogram bisect each other. Mid-point of AC=(2,1) Mid-point Of BD =(2,1)	1 ½ 1/2
24	$(1+\cos A)(1-\cos A) = \frac{3}{4}$ $\therefore \quad 1-\cos^2 A = \frac{3}{4}$ $1-\frac{3}{4} = \cos^2 A$ $\frac{1}{4} = \cos^2 A \Rightarrow \sec^2 A = 4 \Rightarrow \sec A = \pm 2$ OR $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3 \Rightarrow \frac{\cos^2\theta}{\cos^2\theta\left(\frac{1}{\sin^2\theta}-1\right)} = 3$ $\Rightarrow \quad \frac{1}{\text{cosec}^2\theta - 1} = 3 \Rightarrow \frac{1}{\cot^2\theta} = 3$ $\Rightarrow \quad \tan^2\theta = 3 \Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = 60°$ Hence, $\theta = 60°$	1 ½ 1/2 1 ½ 1/2
25	∠OCA = 90° ..[Tangent is ⊥ to the radius through the point of contact ∴ OC ⊥ AB AC = 1/2 (AB) … [⊥ from the centre bisects the chord ⇒ AC = 1/2 (48) = 24 cm In rt. ∆OCA, $OA^2 = OC^2 + AC^2$ … [Pythagoras' theorem $r^2 = (7)^2 + (24)^2$ = 49 + 576 = 625 ∴ r= $\sqrt{625}$ = 25 cm	 ½ ½ ½ 1/2
26	Correct construction	2
27	Correct proof step by step	2

28	Speed of the boat in still water 8 km / hr.	
	Let the speed of the stream be x km/hr. Speed upstream 8xkm/hr.	
	Speed downstream 8xkm/hr.	
	Speed downstream 8xkm/hr.	
	Time taken to go 22 km downstream = 22 /(8 + x)hr	1/2
	Time taken to go 15 km upstream= 15/(8-x)hr	1/2
	According to the question:22 /(8 + x)hr + 15/(8-x)hr =5	½
	$5x^2 - 7x - 24=0$	1
	x=3	½
	OR	
	Let the marks obtained by P in mathematics and science be x and 28- x, respectively. According to the given condition,	1
	(x+3)(28 - x- 4) = 180	1
	x=12or x9	1
	Hence, he obtained 12 marks in mathematics and 16 marks in science or 9 marks in mathematics and 19 marks in science	
29	Taking LHS = $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A)$	
	$= \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right) = \left(\frac{\sin A+\cos A-1}{\sin A}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)$	1
	$= \frac{(\sin A+\cos A)^2-1}{\sin A\cos A} = \frac{\sin^2 A+\cos^2 A+2\sin A\cos A-1}{\sin A\cos A}$	1
	$= \frac{1+2\sin A\cos A-1}{\sin A\cos A} = \frac{2\sin A\cos A}{\sin A\cos A} = 2$	1
30	Correct diagram	1
	Finding angle of elevation	1
	Finding height of kite	1
31	AD is median, So BD=DC.	½
	$AB^2= AE^2+BE^2$	1/2
	$AC2 = AE^2+EC^2$	
	Adding both,	½
	$AB^2+AC^2 = 2AE^2+BE^2+CE^2$	
	$= 2(AD^2-ED^2)+(BD-ED)^2+(DC+ED)^2$	1
	$= 2AD^2-2ED^2+BD^2+ED^2-2BD.ED+DC^2+ED^2+2CD.ED$	
	$= 2AD^2+BD^2+CD^2$	
	$= 2(AD^2+ BD^{2)}$	1/2

32	∠ACO = 90° … [Tangent is ⊥ to the radius through the point of contact In ∆AO’D and ∆AOC ∠O’AD = ∠OAC …(Common ∴ ∠ADO = ∠ACO …[Each 90° ∴ ∆AO’D ~ ∴AOC …(AA similarity AO’/AO=DO’/CO … [In ~ As corresponding sides are proportional r/3r=DO’/CO …[Let AO’ = O’X = OX = r ⇒ AO = r +r+ r = 3r ∴ DO’/ CO=1/3	½ 1 1 1/2
33	 $r = \frac{d}{2} = \frac{4}{2} = 2$ cm ...[∵ $d = \frac{14}{2} - 3 = 4$ Let the side of small square $a = 4$ cm ...[∵ $d = \frac{14}{2} - 3 = 4$ Area of square ABCD = $(\text{Side})^2 = (A)^2$ $= (14)^2 = 196\ \text{cm}^2$ Area of small square PQRS $= (a)^2 = (4)^2 = 16\ \text{cm}^2$ Area of 4 semicircles $= 4 \times \frac{1}{2}\pi r^2$ $= [4 \times 1/2 \times 3.14(2)^2]\ \text{cm}^2$ $= 25.12\ \text{cm}^2$	½ ½ 1 ½ 1/2
34	.Let us suppose that one man alone can finish the work in x days and one boy alone can finish it in y days. ∴ One man’s one day’s work = 1/ x And, one boy’s one day’s work = 1 /y 2 men and 5 boys can finish the work in 4 days. ∴ (2 men’s one day’s work) + (5 boys’ one day’s work)=¼ ⇒2/x+5/y=1/ 4	½ 1

	⇒ 2u + 5v = 1/4 ....(i) Here, 1/x = u and 1/y = v Again, 3 men and 6 boys can finish the work in 3days. ∴ (3 men's one day's work) + (6 boys' one day's work) = ⅓ 3u + 6v = 1 /3 one man alone can finish the work is 18days and one boy alone can finish the work in 36 days.	½ 1/2 1/2
35	Correct diagram Finding result step by step	1 4
36	Let $\theta = 60°$, $r = \frac{14}{2} = 7$ cm Area of shaded region $= ar(\Delta ABC) - 3$ (ar of sector) $= \frac{\sqrt{3}}{4}(\text{side})^2 - 3 \cdot \frac{\theta}{360}\pi r^2$ **...[Area of equilateral Δ = $\frac{\sqrt{3}}{4}$side^2** $= \frac{1.73}{4} \times 14 \times 14 - 3 \times \frac{60}{360} \times \frac{22}{7} \times 7 \times 7$ $= 84.77 - 77 =$ **7.77 cm^2**	1 1 2 1